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Estimating hedged portfolio value-at-risk using the conditional copula: An illustration of model risk

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ABSTRACT

The conventional portfolio value-at-risk model with the assumption of normal joint distribution, which is commonly practiced, exhibits considerable biases due to model specification errors. This paper utilizes the estimation of hedged portfolio value-at-risk (HPVaR) to illustrate the potential model risk due to inappropriate use of the correlation coefficient and normal joint distribution between index spot and futures returns. The results show that HPVaR estimation can be improved by using the conditional copulas and their mixture models to form joint distributions to calculate the optimal hedge ratio. Backtesting diagnostics indicate that the copula-based HPVaR outperforms the conventional HPVaR estimator at both the 99% and the 95% coverage rates. The conventional models obviously underestimate the HPVaR, especially under a 99% coverage rate. We then employ a bootstrap resampling technique to quantify and compare the magnitude of model risk by constructing confidence intervals around HPVaR point estimates. The results suggest that the risk management models should apply a smaller nominal coverage rate (95% instead of 99%) to avoid the model risk mentioned above.

1. Introduction

Value-at-risk (VaR) has become one of the most popular tools for risk measurement. However, it is subject to model risk due to the choice of models, parameter estimation, and their implementation (Brooks & Persand, 2002; Jorion, 1996; Kupiec, 1999; Miller & Liu, 2006; Rich, 2003). Jorion (1996) first indicated that VaR estimates were themselves affected by their sampling variation or "estimation risk." Brooks and Persand (BP) (2002) investigated a number of statistical modeling issues in determining market-based capital risk requirements. They highlighted several potential pitfalls in commonly applied methodologies and concluded that model risk could be serious in VaR calculation.[2]

The conventional portfolio value-at-risk model with the assumption of normal joint distribution, which is commonly used in current practice, exhibits considerable biases due to model specification error (Miller & Liu, 2006). This paper utilizes the estimation of HPVaR to illustrate the potential model risk due to inappropriate use of the correlation coefficient and normal joint distribution between index futures and spot returns.[3] It also considers whether a more accurate HPVaR estimation under the

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[2] They found that when the actual data is fat-tailed, using critical values from a normal distribution in conjunction with a parametric approach can lead to a substantially less accurate VaR estimate than using a nonparametric approach.
[3] The correlation coefficient measures only the "degree" or "level" of dependence, which reflects the overall strength of the relation. However, it fails to model the "structure" of dependence, which describes the manner in which two assets are correlated. In addition, it is neither robust enough for heavily tailed distributions nor adequate for non-linear relationships.

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alternative copula-based joint distributions could be derived.[4] Copulas enable the modeler to construct flexible multivariate distributions exhibiting various patterns of tail behavior, ranging from tail independence to tail dependence, and different kinds of asymmetries. Since the copula approach enables us to identify the dependence structures and to capture the potential nonlinear relation, copulas become alternative measures of correlation (Embrechts, McNeil, & Straumann, 1999; Wang, Chen, & Huang, 2011). This paper employs four types of conditional copulas (Gaussian, Gumbel, Clayton and their mixture) to represent the dependence between index spot and futures returns.

Based on the results of two backtests (the dynamic quantile test, and the distribution and tail forecast test), this study demonstrates that, under all coverage rates (95% and 99%), the copula-based HPVaR model exhibits performance superior to both the conventional constant conditional correlation (CCC) GARCH model (Bollerslev, 1990) and the dynamic conditional correlation (DCC) GARCH model (Tse & Tsui, 2002).[5] The conventional CCC- and DCC-GARCH models obviously underestimate the HPVaR, especially for those under a 99% coverage rate. To further compare the magnitude of model risk (measured by the length of confidence interval around the HPVaR point estimate), this study, similar to that of Christoffersen and Goncalves (2005), employs the bootstrap resampling technique to quantify the magnitude of model risk by constructing confidence intervals around HPVaR point estimates. The result shows that the closer the quantiles are to the mean of the distribution, the smaller the model risk will be for both copula-based and conventional models. Our finding supports the viewpoint of Brooks and Persand (2002)[6] that, to ensure covering virtually all probability losses, the use of a smaller coverage rate (say, 95% instead of 99%) combined with a larger multiplicative factor was preferred.

This paper is organized as follows. Section 2 presents the copula-based GARCH model. Section 3 describes the data and the estimation of HPVaR. In Section 4, we show that copula-based models improve the HPVaR estimation by using the backtests and then in Section 5 we employ the bootstrapping technique to quantify the magnitude of model risk. Finally, the economic implication of model risk is interpreted. Section 6 concludes the paper.

2. The copula-based GARCH model and HPVaR

We assume that the asset returns (index and its corresponding futures) are characterized by a GJR-GARCH(1,1)-AR(1)-t model (Engle & Ng, 1993; Glosten, Jagannathan, & Runkle, 1993; Leeves, 2007). Let $R_{i,t}$ and $h_{i,t}^2$ denote the return of asset i (spot (s) or futures (f)) and its conditional variance for time t, respectively. Ω_{t-1} denotes the information set available at time $t-1$. The GJR-GARCH(1,1)-AR(1)-t model for asset i is defined by[7]:

$$R_{i,t} = a_i + b_i R_{i,t-1} + \varepsilon_{i,t} \tag{1a}$$

$$z_{i,t}\Big|\Omega_{t-1} = \sqrt{\frac{v_i}{h_{i,t}^2(v_i-2)}}\varepsilon_{i,t} \quad z_{i,t}\sim iid\, t_{vi} \tag{1b}$$

$$h_{i,t}^2 = \omega_i + \beta_i h_{i,t-1}^2 + \alpha_{i,1}\varepsilon_{i,t-1}^2 + \alpha_{i,2} s_{i,t-1}\varepsilon_{i,t-1}^2 \tag{1c}$$

$$i\in\{s,f\}$$

with $s_{i,t-1}=1$ when $\varepsilon_{i,t-1}$ is negative and otherwise $s_{i,t-1}=0$. ν is the degree of freedom.

Hsu, Wang, and Tseng (2008) proposed copula-based GARCH models for estimating optimal hedge ratio, and found that they perform more effectively than other dynamic hedging models. In their way, the conditional variance–covariance matrix of residual series from ($\varepsilon_{s,t}$,$\varepsilon_{f,t}$) in Eq. (1a), is denoted by

$$Var\left(\varepsilon_{s,t},\varepsilon_{f,t}\middle|\Omega_{t-1}\right) = \begin{bmatrix} h_{s,t}^2 & h_{sf,t} \\ h_{sf,t} & h_{f,t}^2 \end{bmatrix} \tag{2}$$

[4] VaR applications are somewhat limited in the multivariate case because the portfolio value-at-risk (PVaR) model cannot be stated in a closed form and can only be approximated with complex computational algorithms. Miller and Liu (2006) criticized current PVaR approaches for four reasons. First, the assumption of normal joint distribution is unrealistic despite its convenience. Second, even though nonparametric approaches are not subject to the criticism of distributional assumptions, tail probability estimations based on the empirical distribution function may be poor because the observed outcomes in the tails of leptokurtic distributions are typically sparse. Third, HPVaR estimation based solely on tails of the return distribution rather than on the entire return distribution may incur significant information loss. Finally, significant nonlinear correlations between the portfolio components are evident in previous studies. Although multivariate extreme value theory (EVT) models may help mitigate these problems, it may exhibit undesirable properties and may not prevent substantial HPVaR estimation biases in current practice.

[5] For CCC and DCC models, the conditional multivariate distribution is Gaussian or Student-t. It can capture only the linear relation between index spot and its futures.

[6] Brooks and Persand (2002) found that standard errors could be more severe for the lower percentile of normal return distribution. They suggested that the closer the quantile was to the mean of the distribution, the more accurately VaR could be estimated.

[7] In Enders (2004), the Eq. (1b) can be alternatively expressed as $\varepsilon_{i,t}|\Omega_{t-1}\sim t_{\nu i}(0,h_{i,t}^2)$.