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Model risk and option hedging

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Abstract

This paper presents a theoretical approach to option hedging and valuation when traders are facing model risk. Model risk is restrictively defined as the financial risk resulting from the choice of an approximating model to proxy for the true but ex-ante unknown state space of the underlying security process. A generalized model is defined for estimating the appropriate volatility markup, which is dependent on the noisiness of the volatility estimate over time. Delta neutral hedge portfolios are created using simulated S&P 500 option prices to demonstrate that using a volatility markup in the traditional binomial model reduces model risk.

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Among the many securities that are being traded on financial markets, options possess a very distinctive contractual feature: the writer faces unlimited exposure while the buyer cannot lose more than the premium initially paid. In the early 1970’s Black, Scholes and Merton proposed a new theory to determine the fair price of a European option along with, and perhaps more importantly, a unique trading strategy to perfectly replicate the option’s terminal payoff. From then on, financial institutions have exploited this theoretical foundation to

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write options and hedge their exposure by trading the so-called replicating portfolio. While more sophisticated valuation models are regularly being introduced, effective replication remains of utmost importance for risk management purposes. Thus, these institutions are exposed to model risk, which results from less than perfect replication. If effective replication is not possible, then option writers face the risk of losing substantial amounts of capital with an adverse market evolution.

The arbitrage approach is the most popular technique for option valuation and replication. In this approach, one starts by assuming some exogenous stochastic process for the underlying risky security (typically referred to as the stock). Then, one shows that the payoff of an option written on the risky security can be replicated by trading a specific portfolio composed of the stock and the risk-free security in the appropriate proportions so that valuation and replication have the same payoffs. A major underlying assumption to develop this theory, is that traders have perfect knowledge of the process driving the underlying stock returns. In practice, however, casual empiricism suggests that this is not the case. Traders, for instance, typically implement the binomial model by taking into account current stock price history. Indeed, the binomial tree is routinely "rebuilt" at each time period to match the new current stock price (which systematically differs from the original tree prediction). In addition, it is also often the case that the up and down magnitudes will be reset based on the most recent stock moves. We shall refer to these practices as "dynamic reconstruction." In this paper, we provide theoretical grounding for these ad-hoc practices.

To account for model risk[1], we use the appropriate extension of the standard binomial option-pricing model (SBOPM hereafter). More specifically, we consider an economy where traders are uncertain about the actual state space of the stock's binomial process. Theoretically, of course, this situation can be considered as a particular form of market incompleteness. There is a substantial amount of literature (see Dengler & Jarrow, 1996; Derman & Kani, 1994; Jarrow & Madan, 1995; Rubinstein, 1994 for instance) that deals with completing the market by introducing additional basis securities so that it is again possible to value, by arbitrage, the securities which are not in the pricing basis. This is not the direction that we propose to explore in this paper. Instead, and perhaps more consistent with actual market practices, we assume that traders simply select a specific state space which we shall call the approximating model. Of course, traders also take into account the fact that the approximating model is only an approximation of the unknown state space. Consequently, an optimal choice shall be one where replication errors due to this approximation are minimized. We further show that such a choice is unique and hence propose a methodology to value a call option.

More generally, we are interested in discussing the following issues. First, what is a rational approximating model in this context? Second, how can one formally define and characterize model risk? Third, what kind of hedging strategy will be more effective?

The main contribution of this paper is to explore the limits of the complete market paradigm to value and hedge options. This is certainly an important issue since replication techniques based on the complete market assumption are routinely used by financial institutions to reduce risk when writing options. In addition, we also specialize our general

[1] We acknowledge that our definition of model risk is somewhat restrictive and does not encompass all the aspects of this concept. For a more substantial discussion see Green and Figlewski (1999).