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Econophysical visualization of Adam Smith's invisible hand

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ABSTRACT

Consider a complex system whose macrostate is statistically observable, but yet whose operating mechanism is an unknown black-box. In this paper we address the problem of inferring, from the system's macrostate statistics, the system's intrinsic force yielding the observed statistics. The inference is established via two diametrically opposite approaches which result in the very same intrinsic force: a top-down approach based on the notion of entropy, and a bottom-up approach based on the notion of Langevin dynamics. The general results established are applied to the problem of visualizing the intrinsic socioeconomic force – *Adam Smith's invisible hand* – shaping the distribution of wealth in human societies. Our analysis yields quantitative econophysical representations of figurative socioeconomic forces, quantitative definitions of "poor" and "rich", and a quantitative characterization of the "poor-get-poorer" and the "rich-get-richer" phenomena.

1. Introduction

In our everyday life we speak figuratively of forces driving our economies and our financial markets, forces engaging within our political systems, forces shaping our public opinions, etc. In contrast, in physical sciences the force concept can be sharply defined. Quantitative macroscopic statistical descriptions of complex physical systems can be built up directly from known microscopic forces—a bottom-up approach from the microscopic to the macroscopic. The situation is diametrically opposite in social sciences where we have only a metaphoric description of the underlying "microscopic forces" presumed to govern social systems, while we can often measure the statistical distributions characterizing the "macroscopic states" of social systems.

For example, the distributions of individual wealth in different human societies are well known from detailed measurements [1–6], yet the underlying socioeconomic forces shaping these distributions are both conceptually and quantitatively unclear. A top-down approach is thus needed—a solution to the inverse problem by first defining and then inferring the unknown intrinsic forces quantitatively from macroscopically observable statistical data.

A stationary or quasi-stationary probability distribution within a complex system – e.g., individual wealth within a human society – contains information about the statistical randomness of that distribution within the system. A deep connection exists among information, statistical randomness, and the concept of entropy [7,8]. The connection holds generally, not just in the physical context in which the entropy concept originated [9–14].[1]

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[1] Clausius, in a series of foundational papers culminating in his 1865 monograph, introduced entropy as a macroscopic entity and with it formulated the second law of thermodynamics—that entropy always remains unchanged or increases [9]. Maxwell pointed out in an 1867 letter to Tait – by inventing an eponymous demon apparently capable of violating the second law of thermodynamics – that full understanding of the entropy concept had not yet been

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In this paper we extend that general connection beyond entropy to a concept analogous to that of the free energy of statistical thermodynamics [15]. The extension allows us to introduce into general non-physical contexts the concept of a deterministic or systematic "intrinsic force" which is analogous to a physical force. This intrinsic force acts on a quantity, a measurable microscopic property of a complex system, shaping its macroscopic probability distribution function (PDF). We show how the intrinsic force can be extracted quantitatively up to a scale factor from the macroscopic PDF when the latter is known, thus solving the top-down inversion problem.

Fluctuations in the microscopic property imply as well the existence of a stochastic force. Both forces – the deterministic and the stochastic – act together to generate the time dependence of the quantity within an equation of motion analogous to the equation of motion of a physical quantity. In simple cases the equation of motion takes the form of a Langevin equation [16,17]. Absent the deterministic force, the stochastic force would drive the distribution to statistical uniformity. Absent the stochastic force, the deterministic force would drive the distribution to statistical degeneracy—the mode of a unimodal PDF, or a local mode of a multimodal PDF, which become sharp once the stochastic force is removed. The observed macroscopic PDF is thus shaped by the interplay of the deterministic and stochastic forces and manifests their macroscopic balance. Moreover, the scale factor mentioned above quantifies the relation between the strengths of the deterministic and stochastic forces.

Within this general conceptual structure we infer the existence of a universal, systematic, intrinsic socioeconomic force underlying the distribution of wealth in human societies [1–6]. This socioeconomic force quantifies the familiar observation that there is less resistance to the rich getting richer than to the poor doing so [18,19]. Thus, we present a quantitative econophysical visualization of *Adam Smith's invisible hand* [20,21] – in the context of the distribution of wealth in human societies – and implicitly extend the applicability of the invisible-hand notion to complex systems in general. Moreover, this socioeconomic force yields universal quantitative definitions of "poor" and "rich", and a quantitative measure of the socioeconomic distance between the poor and the rich in a human society.

The paper is organized as follows. Section 2 defines the microscopic deterministic force and shows how it can be inferred from a known macroscopic PDF via a top-down approach. Section 3 posits the existence of both deterministic and stochastic microscopic forces acting within an equation of motion, and shows that this bottom-up approach leads to the same results as the top-down approach of Section 2. The physical formalism of Sections 2 and 3 is appropriate for stochastic systems with elemental additive structures; however, humans respond to relative – rather than to absolute – changes in quantities. Section 4 thus establishes a formalism which is appropriate for systems with elemental multiplicative structures, e.g., socioeconomic systems. Applying the formalism of Section 4 to the distribution of wealth in human societies, Section 5 presents an econophysical visualization of Adam Smith's invisible hand, and an econophysical quantification of the "poor-get-poorer" and the "rich-get-richer" phenomena. Using the econophysical visualization of Adam Smith's invisible hand, Section 6 introduces the universal quantitative definitions of "poor", "rich", and the socioeconomic distance between them. Section 7 concludes the paper with a short discussion.

2. From probability distributions to forces: a top-down approach

Consider a system comprised of n individual entities labeled by the index $i = 1, \ldots, n$, each of which has associated with it a real quantity x_i scaled to be dimensionless. The quantities take values in the range $l < x_i < u$, where l and u are, respectively, the range's lower and upper bounds. Were the system a human socioeconomic system, for example, x_i could be the wealth of the ith individual within it. The "microscopic" state (microstate) of the system is specified by the discrete set of values $\{x_i\}_{i=1}^{n}$.

Suppose that n is large enough that the system can be accurately represented as continuously distributed. The "macroscopic" state (macrostate) of the system is then characterized by the PDF $\Phi(x)$ $(l < x < u)$ to which a number of microscopic states correspond. The individuals x_i vary with time in response to an underlying dynamical process. The microstate of the system correspondingly changes, as does the macrostate, $\Phi(x)$. Observation of $\Phi(x)$ may be made at a particular time or constructed as an average over a period of observation. In either case, for the classes of systems under consideration which have dynamics stationary or quasi-stationary in time, the result would be the most likely PDF $\Phi_*(x)$—that corresponding to the largest number of microstates, all others being of negligible likelihood for macroscopically large n.

Given the PDF $\Phi(x)$, the randomness of the macroscopic state of the system, or equivalently the information contained in $\Phi(x)$, is quantified by the *Boltzmann–Gibbs–Szilard–Shannon* entropy [7,8]:

$$\mathcal{S}(\Phi) = -\int_l^u \Phi(x) \ln(\Phi(x))\, \mathrm{d}x. \tag{1}$$

More importantly, this entropy is monotonically related to the likelihood of the system's macrostate. Indeed, it was Boltzmann's great insight that – in systems with n macroscopically large – the macrostate with the largest number of

achieved [10]. In 1877 Boltzmann laid clear the microscopic origin of the entropy concept [11]. Gibbs, in work leading up to his magisterial monograph on statistical mechanics of 1902, expressed entropy explicitly in probabilistic terms [12]. Szilard resolved the Maxwell's demon paradox by recognizing explicitly the connection between information and entropy, which was implicit in Boltzmann's and Gibbs' formulations of the entropy concept; Szilard pointed out that when the destruction of information by the demon was accounted for, the second law of thermodynamics held [13]. In what for our purposes can be regarded as the culmination of the development of the entropy concept, Shannon initiated entropy-based information theory in 1948 [14].