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Existence of equilibrium in common agency games with adverse selection ☆

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ABSTRACT

We establish the existence of subgame perfect equilibria in general menu games, known to be sufficient to analyze common agency problems. Our main result states that every menu game satisfying enough continuity properties has a subgame perfect equilibrium. Despite the continuity assumptions that we make, discontinuities naturally arise due to the absence, in general, of continuous optimal choices for the agent. Our approach, then, is based on (and generalizes) the existence theorem of [Simon, L., Zame, W., 1990. Discontinuous games and endogenous sharing rules. Econometrica 58 (4), 861–872] designed for discontinuous games.

1. Introduction

In many important examples in multi-contracting mechanism design, several principals (attempt to) contract with a common agent to influence her choice. Such a common agency model has been the focus of much of the recent research in incentive theory.[2]

In the common agency model, principals offer a menu of contracts to the agent, who chooses one contract from those being offered. Although one could imagine more general communication channels between the principals and the agent, Martimort and Stole (2002), Page and Monteiro (2003), Peters (2001, 2003) have shown that such a procedure of offering menus of contracts is enough to characterize the set of equilibrium allocations. In fact, as Martimort (2006) points out, "what matters per se is not the kind of communication that a principal uses with his agent but the set of options that this principal makes available to the agent." This result, known as the delegation principle, implies that the common agency problem can be analyzed through a menu game.

However, in order for the delegation principle to be meaningful, an equilibrium must exist. In this paper, we present a solution to this problem by establishing a general existence theorem for menu games.

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[2] See Martimort (2006) for a survey.

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A menu game is defined as follows.[3] First, the agent's type is drawn from a commonly known distribution. Then, the principals simultaneously choose a menu of contracts (defined as a closed subset of a compact contract space) without observing the agent's type. Finally, the agent chooses one contract (or one contract of each principal), knowing her type and the menus offered by the principals.

The subgame perfect equilibria of a menu game can be easily described by noting that a strategy for the agent induces a normal-form game between the principals.[4] In fact, this is a game where each principal has the set of all possible menus as his own pure strategy set and his payoff is determined by the choice of all principals together with the agent's strategy. Thus, a subgame perfect equilibrium consists of an optimal strategy for the agent and a Nash equilibrium for the normal-form game induced by such strategy.

The problem of existence of a subgame perfect equilibrium would then be trivial if there were a continuous (in the principals' actions) optimal strategy for the agent. Indeed, the normal-form game induced by such strategy would be continuous and standard existence theorems would apply. The difficulty with the existence of equilibrium is that, in general, no optimal strategy for the agent is continuous even if the agent has a continuous utility function, a compact action space and a continuous choice correspondence.

Nevertheless, in a sufficiently continuous menu game (e.g., a menu game with continuous payoff functions for the principals and the agent, as well as with compact choice sets and a continuous choice correspondence for the agent) discontinuities can only arise as a result of a discontinuous strategy for the agent. However, such discontinuities create no problem for the existence of equilibrium. Indeed, it follows from the above description of a subgame perfect equilibrium that we can regard the family of normal-form games induced by the agent's strategies as a game with an endogenous sharing rule as in Simon and Zame (1990) and, therefore, use their existence theorem to establish the existence of subgame perfect equilibria in menu games. In fact, a vector of menus defines a subset of payoffs for the principals, each of which corresponds to a particular strategy of the agent. This clearly defines a correspondence from principals' strategies into payoffs as required for a game with an endogenous sharing rule.

In order to use Simon and Zame's theorem in our setting, we need to generalize it to allow the payoff correspondence to depend, in a measurable way, on the agent's type. Using an approach similar to that of Simon and Zame (1990), we show that any such generalized game with endogenous sharing rules has a solution. This extension is nontrivial because the agent's type space is not assumed to be compact, but merely complete and separable and the payoff correspondence is not assumed to be jointly upper hemi-continuous, but only upper hemi-continuous in the principals' choices and jointly measurable.

Once the above generalization is accomplished, we can easily obtain a subgame perfect equilibrium from a solution. In fact, the Borel implicit function lemma of Furukawa (1972) shows that when the payoff correspondence is the composition of players' payoff functions with some correspondence (interpreted as the optimal choice correspondence of players whose behavior is not explicitly modeled), then every measurable selection from the payoff correspondence can be obtained as the composition of players' payoff function and a measurable selection of this other correspondence. Combining our generalization with this result, we show that every menu game satisfying enough continuity properties has a subgame perfect equilibrium.

Independently, Monteiro and Page (in press) have shown that the common agency game can be reduced to a game with an endogenous sharing rule, to which the existence theorem of Simon and Zame (1990) can be applied. They then show that the agent's equilibrium strategy can always be chosen to have a support no larger than then the number of principals plus one and, when the agent's type space is nonatomic, that it can be taken to be a pure strategy.

The existence of equilibrium in menu games has also been addressed by Page and Monteiro (2003) and Monteiro and Page (2008). The main difference between our approach and the approach taken in these papers arises because they fix exogenously a tie-breaking rule for the agent (to be used when she is indifferent between several contracts) and then they establish simultaneously the existence of an optimal strategy for the agent satisfying the tie-breaking rule and equilibrium strategies for the principals. In contrast, we proceed by determining simultaneously the equilibrium strategies for the principals and for the agent without fixing exogenously any tie-breaking rule.

As Simon and Zame (1990) have pointed out, endogenizing the tie-breaking rule simplifies the existence problem considerably. Due to this simplification, our existence result enables us to dispense with several of the assumptions made in Monteiro and Page (2008), obtaining as a result a richer economic model that allows for: (1) a more general contract space, allowing, in particular, for non-exclusive contracts, (2) more general payoff functions for the principals that, in particular, can depend on the menu of contracts being offered and (3) more general utility functions for the agent.

Furthermore, at a technical level, our result dispenses with the equicontinuity assumption on the agent's utility function used by Monteiro and Page (2008). However, in contrast with their result, ours requires continuous payoff functions for the principals whereas the existence result of Monteiro and Page (2008) allows for upper semi-continuous payoff functions that are quasi-linear.

[3] Our formalization is based on Page and Monteiro (2003).

[4] Although researchers in this field typically focus on perfect Bayesian equilibrium, we note that the set of subgame perfect equilibria of a menu game coincides with the set of its perfect Bayesian equilibria.