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A general existence result for the principal-agent problem with adverse selection

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Abstract

Considering adverse selection with a continuum of types, a general characterization of implementability in terms of *h*-convexity is provided. This enables to write the principal's program as a variational problem with *h*-convexity constraint for which existence of a solution is proved. The class of models considered here is large since the dimension of the parameter may differ from that of the contract and no structural assumption of single-crossing type is required. In particular calculus of variations problems for which admissible functions are convex ones or convex solutions to multi-time Hamilton–Jacobi equations are particular cases of the problems studied below.

Keywords: Principal-agent problem; Adverse selection; Incentive-compatibility

1. Introduction, economic motivation

Studying principal-agent models with adverse selection is usual in economic contract theory. In such models, agents' preferences are given by a parametrized utility function

$$V(\theta, x, t) = h(\theta, x) + t$$

where θ (the *parameter*) is an individual unobservable characteristic of the agent, belonging to Ω, the set of types, $x \in A$ (in all the following $A = \boldsymbol{R}_+^N$) is the agent's *action* and *t* is a scalar monetary transfer. The previous expression means then that if an agent with type θ has to make action *x* and receives transfer *t* his welfare level is $h(\theta, x) + t$.

An agent or an institution (government, firm, social planner, etc. called the principal aims to contract with this population of agents, i.e. to give them incentives to make actions by means of a monetary transfer. The principal is unable yet to observe agents' individual characteristic θ. Therefore, the principal seeks to build *incentive-compatible* (or *incentive-proof*)

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contracts. An incentive-compatible contract is a pair of mappings (x, t) from Ω to $A \times \boldsymbol{R}$ such that for each type θ it is optimal to choose ($x(\theta)$, $t(\theta)$). In other words, a contract ($x(\cdot)$, $t(\cdot)$) is incentive-compatible if and only if for every type θ the following holds:

$$h(\theta, x(\theta)) + t(\theta) = \max_{\theta' \in \Omega} h(\theta, x(\theta')) + t(\theta').$$

This condition is usually known as the incentive-compatibility condition.

At this stage, two questions arise:

1. How can one characterize incentive-compatible contracts in a *general* and *tractable* way?
2. Do there exist incentive-compatible contracts that maximize the principal's profit and how can the principal determine such optimal contracts?

Concerning (1), only two cases have been studied in depth in the literature: the case where θ is scalar (under an additional structural assumption on h usually called *single-crossing condition*), and the case where h is linear with respect to θ. Since a satisfying answer to (1) is a minimal preamble before studying (2), optimality of incentive-compatible contracts has been studied in details only in those two cases.

In the scalar case, the principal's program is equivalent to a calculus of variations problem for which admissible functions are nondecreasing ones. This problem and its connection with adverse selection theory was first studied in the framework of monopoly pricing by Mussa and Rosen (1978). For such problems, existence and characterization of the solutions have been studied by Rochet (1989) in the convex case and by Carlier (2000a) in the nonconvex one.

In the linear case, the principal's program leads to calculus of variations with convexity constraint, see Choné and Rochet (1998). Recently, Euler–Lagrange equation for such problems has been established by Lions (1998), see also Carlier (2000a,b) for a different approach.

The aim of this paper is to study general parametrized preferences $h(\theta, x)$. In Section 2, incentive-compatible contracts are characterized in terms of some envelope property called h-convexity. In Section 3, using this characterization the principal's problem is written as a variational problem with h-convexity constraint. Existence results for nonstandard variational problems of that kind are then established in Section 4 and several examples of applications are given in Section 5.

These results rely very much on h-convexity, h-subdifferentiability and properties of h-convex potentials play a crucial role in the proofs. These notions have recently been used fruitfully to solve almost explicitly in some cases Monge's mass transportation problem (see Gangbo and Mc Cann (1997), Rachev and Ruschendorf (1998), following the work of Brenier (1991)). Thus, it is worth giving an example of how these concepts can be used in the framework of mathematical economics.

It should be pointed out that the link between incentive-compatible contracts and h-cyclical monotonicity, that would not be used here, had been established by Rochet (1987), see also Levin (1997).

After this work was completed, I read Monteiro and Page's paper (1998) where similar general existence results for the principal-agent problem with adverse selection are proved. However, the problem they address is different from the one studied here