Journal of Mathematical Economics 55 (2014) 1–3

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

A note on the equilibrium theory of economies with asymmetric information

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ARTICLE INFO

Article history:
Received 20 April 2014
Received in revised form
31 July 2014
Accepted 25 August 2014
Available online 16 September 2014

Keywords:
Asymmetric information
General equilibrium theory
Infinite dimensional commodity spaces
Individually rational utility set

ABSTRACT

In this note, we give an equilibrium existence theorem for exchange economies with asymmetric information and with an infinite dimensional commodity space. In our model, we assume that preferences are represented by well behaved utility functions, the positive cone has a non empty interior and the individual rational utility set is compact. Our result complements the corresponding one in Podczeck and Yannelis (2008), in the sense that is applicable to commodity spaces in which the order intervals are (possibly) not compact with respect to any Hausdorff linear topology.

1. Introduction

In a seminal paper, Podczeck and Yannelis (2008) establish equilibrium existence results in the setting of economies with asymmetric information and with infinitely many commodities. The assumptions on the model allow for very general preferences, i.e., preferences need not be transitive or complete. The commodity spaces treated include any ordered Hausdorff locally convex space (E, τ) such that the positive cone has a non empty interior and the order intervals are compact with respect to some Hausdorff linear topology η.

The latter compactness condition is found in many commodity spaces, for instance, the order intervals of any dual Banach lattice are compact with respect to the w^*-topology. Nonetheless, there exist classical commodity spaces that do not satisfy this assumption. In particular, it is proved in Xanthos (in press, Proposition 2.2) that ordered vector spaces which are not monotone order complete (e.g. $C[0, 1]$, c, $C^1[0, 1]$) fail to have compact intervals under any Hausdorff linear topology. As a result the equilibrium existence theorems in Podczeck and Yannelis (2008) cannot be applied for economies defined on these commodity spaces.

In this note, we solve the equilibrium existence problem for this class of commodity spaces. In particular, we show that Theorem 1 in Podczeck and Yannelis (2008) remains valid, if we assume that the preferences are represented by utility functions and substitute the compactness assumption of the order intervals by requiring that the individually rational utility set is compact. This assumption goes back to Mas-Colell (1986) and is a standard alternative compactness assumption in equilibrium theory (see in Allouch and Florenzano (2004), Brown and Werner (1995), Cheng (1991), Chichilnisky and Heal (1993), Dana et al. (1997)). We refer the reader to Aliprantis et al. (1990) for examples of utility functions in $C[0, 1]$ that satisfy this assumption. In our proof, we use a different approach than the one in Podczeck and Yannelis (2008), that is the classical decentralization method (see e.g. in Florenzano (2003)). In particular we apply, a result of Allouch and Florenzano (2004), to ensure the non emptiness of the fuzzy core and we obtain an equilibrium by decentralization. It is worth mentioning here that this approach can also give an alternative proof of Theorem 1 in Podczeck and Yannelis (2008). A crucial step in the decentralization process is a fundamental property of economies with differential information that was highlighted in Podczeck and Yannelis (2008). That is, the total consumption space can be written as a topological direct sum of some subspaces that contain the "essential" information of the consumers.

For a detailed discussion about economies with differential information and the economical interpretation of the assumptions in our model, we refer the reader to Podczeck and Yannelis (2008) and the references inside.

2. The model and the result

In this paper we will adopt the terminology of Podczeck and Yannelis (2008). Let E be an ordered vector space endowed with a

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http://dx.doi.org/10.1016/j.jmateco.2014.08.006
0304-4068/

Hausdorff locally convex topology such that E_+ is closed. Let Ω be a non-empty finite set of states of nature. Given a partition $\mathcal{P}$ of Ω, we will say that an element $x \in E^\Omega$ is $\mathcal{P}$-measurable if for any $S \in \mathcal{P}$ we have that $x(s) = x(s')$ for any $s, s' \in S$. A differential information economy $\mathcal{E}$ with finitely many agents and commodity space E^Ω is a family $\mathcal{E} = [(\mathcal{H}_i, u_i, w_i)]$, where $I = \{1, \dots, n\}$ is the finite set of agents and for each agent i,

- the private information partition of Ω is denoted by $\mathcal{H}_i$,
- the consumption set is denoted by X_i and is defined as follows,

  $X_i = \{x \in E_+^\Omega \mid x \text{ is } \mathcal{H}_i\text{-measurable}\}$.
- the initial endowment is denoted $w_i \in X_i$ and we assume that $\sum_{i\in I} w_i \neq 0$,
- the utility function is denoted $u_i : X_i \to \mathbb{R}$ and we assume that $u_i(w_i) = 0$.

In the following we assume that E^Ω is equipped with the product topology. For each utility function u_i we associate the strict preference relation $P_i : X_i \to 2^{X_i}$, which is defined as follows

$$P_i(x) = \{y \in X_i \mid u_i(y) > u_i(x)\}.$$

A **feasible allocation** for the economy $\mathcal{E}$ is a list $\mathbf{x} = (x_i)_{i\in I}$ where $x_i \in X_i$ for each $i \in I$ and $\sum_{i\in I} x_i = \sum_{i\in I} w_i$. We denote the set of all feasible allocations of the economy $\mathcal{E}$ with $\mathcal{A}(\mathcal{E})$.

The individually rational utility set is the following set

$$\mathcal{U} = \{v = (v_i)_{i=1}^n \in \mathbb{R}_+^n \mid \exists \mathbf{x} = (x_i) \in \mathcal{A}(\mathcal{E}) \mid 0 \le v_i \le u_i(x_i) \ \forall i \in I\}.$$

The **fuzzy core** $\mathcal{C}^F(\mathcal{E})$ of the economy $\mathcal{E}$ is the set of all $\mathbf{x} = (x_i) \in \mathcal{A}(\mathcal{E})$ such that there is no $t = (t_i)_{i\in I} \in [0, 1]^n \setminus \{0\}$ and no $\mathbf{x}^t = (x_i^t) \in \prod_{i\in \mathrm{supp}(t)} X_i$ such that

$$\sum_{i\in \mathrm{supp}(t)} t_i x_i^t = \sum_{i\in \mathrm{supp}(t)} t_i w_i \quad \text{and } u_i(x_i) < u_i(x_i^t)$$

for each $i \in \mathrm{supp}(t)$.

Note that any element of the fuzzy core is an individually rational and Pareto optimal allocation (see e.g. in Florenzano (2003)). A **non trivial quasi-equilibrium** for $\mathcal{E}$ is a pair $(\mathbf{x}, p)$ where $\mathbf{x} \in \mathcal{A}(\mathcal{E})$ and $p \in E^{\Omega,*}$ is a price system such that

- $p(x_i) = p(w_i)$ for each $i \in I$,
- $y \in P_i(x_i) \Rightarrow p(y) \ge p(w_i)$ for each $i \in I$,
- $p(w_i) > \inf\{p(y) \mid y \in X_i\}$ for some $i \in I$.

To obtain our equilibrium existence result, we will make the following assumptions on the economy $\mathcal{E}$[1]

(A1) The positive cone E_+ has a non empty interior.
(A2) For each $i \in I$, u_i is quasi-concave.
(A3) The individually rational utility set $\mathcal{U}$ is compact.
(A4) If $\mathbf{x} = (x_i)$ is a feasible and individually rational allocation then u_i is lower semicontinuous at x_i for every $i \in I$.
(A5) If $\mathbf{x} = (x_i) \in \mathcal{C}^F(\mathcal{E})$ then $x_i \in \overline{P_i(x_i)}$ for every i.
(A6) $\sum_{i\in I} w_i(s)$ is an interior point of E_+ for each $s \in \Omega$ and there exists a feasible allocation $\mathbf{z} = (z_i)$ such that for each $i \in I$ and each $s \in \Omega$, $\lambda \sum_{i\in I} w_i(s) \le z_i(s)$ for some real number $\lambda > 0$.

The main result of our paper is the following equilibrium existence result.

[1] The following assumptions are the appropriate analogue of the assumption (A1)–(A7) in Podczeck and Yannelis (2008), for preferences represented by utility functions. The notable difference is that the compactness assumption (A3) we use here is strictly weaker than the corresponding assumption (A5) in Podczeck and Yannelis (2008).

Theorem 2.1. *Under assumptions* (A1)–(A6)*, the economy $\mathcal{E}$ has a non trivial quasi-equilibrium* $(\mathbf{x}, p)$*, where* $\mathbf{x} \in \mathcal{C}^F(\mathcal{E})$.

As a corollary of the above result, we get that if additionally the economy $\mathcal{E}$ satisfies a suitable irreducibility condition (see relevant discussion in Florenzano (2003, Section 2.3)), then $\mathcal{E}$ has an equilibrium.

3. The proof

Our proof is based on the main result of Allouch and Florenzano (2004), where the authors proved the existence of Edgeworth equilibria for exchange economies with consumption sets which are (possibly) unbounded below (see Theorem 3.1 and Remark 3.4 in Allouch and Florenzano (2004)). In our setting, the following result holds.

Theorem 3.1 (*Allouch and Florenzano, 2004*)**.** *Under assumptions* (A2)–(A4)*, the fuzzy core $\mathcal{C}^F(\mathcal{E})$ of the economy $\mathcal{E}$ is nonempty.*

Before we proceed to our proof, we note that the decentralization method we use here can give an alternative proof of Theorem 1 in Podczeck and Yannelis (2008), if we use Proposition 5.2.3 in Florenzano (2003), instead of the above theorem.

We give below some notations and mathematical preliminaries, that we will use in the following. We will use the symbol $\ge$ for the standard ordering of E^Ω, that is $x \ge 0$ if and only if $x(s) \in E_+$ for each $s \in \Omega$. If P is a cone of E^Ω, then we will denote the ordering induced by P with the symbol $\ge_P$ and we will write $x \ge_P 0$ if and only if $x \in P$. We say that $x_0 \in P$ is an order unit for $P - P$ with respect to the ordering induced by P, if for any $y \in P$, there exists $n \in \mathbb{N}$ such that $y \le_P nx_0$. We denote with $M_i = X_i - X_i$ the individual consumption space, with $\mathcal{C} = \sum_{i\in I} X_i$ the total consumption set, and $M = \mathcal{C} - \mathcal{C}$ the total consumption space. For any set A in E^Ω, we denote the closure and the interior of A with respect to the product topology by $\overline{A}$ and $\mathrm{int}(A)$ respectively. If Y is a subset of E^Ω such that $A \subseteq Y$, we denote with $\mathrm{int}_Y A$ the interior of A with respect to the relative topology on Y. For each $S \subseteq \Omega$ we denote with $1_S \in \mathbb{R}^\Omega$ the indicator function. It is useful to note that under this notation, each consumption space M_i can be written in the following way

$$M_i = \left\{ \sum_{S\in \mathcal{H}_i} e_S 1_S \mid e_S \in E \right\}.$$

Lemma 3.2. *Assume that* (A1) *holds. Then for any $i \in I$ we have that*

(i) *X_i is a closed cone with* $\mathrm{int}_{M_i} X_i \neq \emptyset$.
(ii) *If A is a subset of X_i such that* $\mathrm{int}_{X_i} A \neq \emptyset$*, then* $\mathrm{int}_{M_i} A \neq \emptyset$.
(iii) *If* (A6) *holds, then there exists a feasible allocation $\mathbf{z} = (z_i)$ such that $\sum_{i\in I} z_i$ is an order unit of M with respect to the ordering induced by $\mathcal{C}$.*

Proof. (i) By the definition of the consumption set, we have that X_i is a closed cone. By (A1) there exists $e \in \mathrm{int}(E_+)$, hence there exists W open neighborhood of zero in E, such that $e + W \subseteq E_+$. Then note that $e1_\Omega + W^\Omega \cap M_i \subseteq X_i$, thus $\mathrm{int}_{M_i} X_i \neq \emptyset$ for any $i \in I$.

(ii) Fix some $i \in I$ and let $Q = \{x \in X_i \mid x \in \mathrm{int}_{M_i} X_i\}$ be the set of interior points of X_i with respect to the relative topology on M_i. By (i) it follows that $Q \neq \emptyset$, we will show next that $\overline{Q} = X_i$. Since X_i is closed it follows that $\overline{Q} \subseteq X_i$. Suppose that there exists $y \in X_i \setminus \overline{Q}$, it is easy to verify that Q is convex, thus there exists a non zero continuous functional $f \in E^{\Omega,*}$ that strongly separates $\overline{Q}$ and y (see e.g. in Aliprantis and Border (2007, Theorem 5.80)). Let $a \in \mathbb{R}$ such that $f(z) > a > f(y)$ for each $z \in \overline{Q}$. Note that $\lambda Q \subseteq Q$ for each $\lambda > 0$, thus $a \le 0$ and there exists $z_0 \in Q$ such that $f(z_0 + y) < a$. On the contrary we have that $Q + X_i = Q$, therefore $f(z_0 + y) > a$, a contradiction. Since $\mathrm{int}_{X_i} A \neq \emptyset$ there exists some