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Robust efficiency in mixed economies with asymmetric information

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ABSTRACT

In this paper, we study robustly efficient allocations in a pure exchange economy. Answering a question of Hervés-Beloso and Moreno-García (2008), we present an extension of their main result to an asymmetric information mixed economy whose commodity space is an ordered separable Banach space having an interior point in its positive cone.

1. Introduction

In the literature of modern economics, how to characterize competitive allocations of an economy has been an interesting topic for many authors. Particularly, since Aumann (1964) published his seminal work on the equivalence of the core and the set of Walrasian allocations for an atomless economy with a continuum of agents and finitely many commodities, many authors have become interested in exploring the relationship between the core and the set of Walrasian allocations in various types of economies. In the past fifty years, Aumann's equivalence theorem has been sharpened and extended in several directions. First, three notes in the same issue of *Econometrica* by Schmeidler (1972), Grodal (1972) and Vind (1972) respectively gave sharper interpretations to Aumann's theorem as a characterization of perfect competition. According to Vind (1972), if some coalition blocks an allocation then there is a blocking coalition with any measure less than the measure of the grand coalition. Thus, for an atomless economy with a finite-dimensional commodity space, the set of Walrasian allocations coincides with the set of allocations that are not blocked by coalitions of arbitrarily given measure less than that of the grand coalition. Second, Shitovitz (1973) extended Aumann's equivalence theorem to mixed economies, where he proved that if there are at least two large agents in an economy and if they are similar to each other, i.e., if they have the same initial endowment and the same preferences, then core allocations are competitive. Third, Aumann's equivalence theorem was extended to the framework of uncertainty with asymmetric information first introduced by Radner (1968). In this framework, Yannelis (1991) introduced the concept of the private core which is based on individual measurability requirement. Under free-disposal, Einy et al. (2001) proved that for an atomless economy with finitely many commodities and asymmetric information, if the economy is irreducible, then the private core and the set of Walrasian expectations allocations coincide. Recently, Angeloni and Martins-da-Rocha (2009) have further showed that the same conclusion holds without free disposal if the prices are allowed to take negative values.

On the other hand, for an atomless economy with finitely many commodities, Hervés-Beloso and Moreno-García (2008) gave a different characterization of Walrasian allocations. Instead of using infinitely many coalitions, they only used the veto power of the grand coalition, but exercised in a family of economies obtained by perturbing the agents' initial endowments. Precisely, Hervés-Beloso and Moreno-García proved that in a continuum economy with finitely many commodities the set of Walrasian allocations are those that are non-dominated in any economy obtained by a slight perturbation of the real endowments of the agents belonging to either arbitrarily small coalitions, arbitrarily large coalitions, or coalitions of a given measure. This is a kind of core-Walras equivalence theorem in which one does not consider the veto power of infinitely many coalitions but the veto power of a single coalition in infinitely many economies. As applications, this theorem leads one to obtain the first and second welfare theorems as easy corollaries. In addition, applying this theorem to a continuum economy with n different types of agents, one

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obtains the characterization of the Walrasian allocations showed in Hervés-Beloso et al. (2005a) as a particular case. In the last section of Hervés-Beloso and Moreno-García (2008), they pointed out that this result can be extended to an asymmetric information economy with the space of real bounded sequences as the commodity space, and then they asked whether this result can be extended to economies with other commodity spaces.

The main purpose of this paper is to tackle the afore-mentioned problems arising in Hervés-Beloso and Moreno-García (2008). The paper is confined to pure exchange and mixed economies with asymmetric information and having infinitely many commodities. The paper is organized as follows. In Section 2, we give a general description on our model. In Section 3, we provide several technical results on the private blocking power of coalitions in an atomless economy with asymmetric information. Our main result of this paper is given in Section 4, where we give an extension of the main result in Hervés-Beloso and Moreno-García (2008) to an asymmetric information mixed economy whose space of agents is a complete finite positive measure space and whose commodity space is an ordered separable Banach space having an interior point in its positive cone. Thus, the problem mentioned by Hervés-Beloso and Moreno-García is addressed. In Section 5, we summarize our main results and give some open questions.

2. A mixed model with asymmetric information

2.1. The model

We consider a mixed model of a pure exchange economy $\mathscr{E}$ with asymmetric information. There is a finite space of states of nature, denoted by Ω. The economy extends over two time periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$, there is uncertainty over the states and agents make contracts that are contingent on the realized state at $\tau = 1$. Let the space of agents be a measure space (T, Σ, μ) with a complete, finite and positive measure μ, where T is the set of agents and Σ is a σ-algebra of measurable subsets of T. Throughout the paper, the commodity space is assumed to be an ordered Banach space Y whose positive cone Y_+ has an interior point and Y_+ is the *consumption set* in every state $\omega \in \Omega$ for each agent $t \in T$. Some classical ordered Banach spaces having interior points in their positive cones are: (i) $\mathbb{R}^n$: the Euclidean space; (ii) ℓ_∞: the space of real bounded sequences with the supremum norm; (iii) $L_\infty(X, \mathscr{M}, m)$: the space of essentially bounded, measurable functions on a measure space $(X, \mathscr{M}, m)$ with the essential supremum norm; (iv) $C[a, b]$: the space of real-valued continuous functions on the closed interval $[a, b]$ with the supremum norm; (v) c: the space of convergent sequences endowed with the supremum norm. Note that Banach spaces (i), (iv) and (v) are separable. Each agent $t \in T$ is associated with his *characteristics* $(\mathscr{F}_t, U_t, a(t, \cdot), q_t)$, where $\mathscr{F}_t$ is the σ-algebra generated by a partition Π_t of Ω representing the *private information* of t; $U_t : \Omega \times Y_+ \to \mathbb{R}$ is the *state-dependent utility function* of t; $a(t, \cdot) : \Omega \to Y_+$ is the *random initial endowment* of t and q_t is a probability measure on Ω giving the *prior* of t.

For each $\omega \in \Omega$, one defines $\psi_\omega : T \times Y_+ \to \mathbb{R}$ by $\psi_\omega(t, x) = U_t(\omega, x)$ for all $(t, x) \in T \times Y_+$. For each $\omega \in \Omega$, ψ_ω is assumed to be Carathéodory, i.e., $\psi_\omega(\cdot, x)$ is measurable for all $x \in Y_+$ and $\psi_\omega(t, \cdot)$ is norm-continuous for all $t \in T$. For any $n \geq 1$, the $(n-1)$-simplex of $\mathbb{R}^n$ is defined as

$$\Delta^n = \left\{ x = (x_1, \ldots, x_n) \in \mathbb{R}^n_+ : \sum_{i=1}^{n} x_i = 1 \right\}.$$

Define a function $\varphi : (T, \Sigma, \mu) \to \Delta^{|\Omega|}$ by $\varphi(t) = q_t$ for all $t \in T$. Here, φ is assumed to be measurable with respect to the Borel structure on $\Delta^{|\Omega|}$. We call $\mathscr{E}$ a *standard economy* if the above two assumptions on ψ_ω and φ are satisfied. In the rest of the paper, the economy $\mathscr{E}$ is assumed to be standard and we simply call it an economy without any confusion. A function $x : \Omega \to Y_+$ is interpreted as a *random consumption bundle* in $\mathscr{E}$. The *ex ante expected utility* of an agent t for a given random consumption bundle x is defined by $V_t(x) = \sum_{\omega \in \Omega} U_t(\omega, x(\omega)) q_t(\omega)$. Note that the function $(t, x) \mapsto V_t(x)$ is Carathéodory.

The order on Y is denoted by $\leq$ and the symbol $x \gg 0$ means that x is an interior point of Y_+. Let $Y_{++} = \{x \in Y_+ : x \gg 0\}$. Any set $S \in \Sigma$ with $\mu(S) > 0$ is called a *coalition* of $\mathscr{E}$. If S and S' are two coalitions of $\mathscr{E}$ with $S' \subseteq S$, then S' is called a *sub-coalition* of S. Let $L_1(\mu, Y)$ be the set of all Bochner integrable functions from T into Y_+ and $L_t = \{x \in (Y_+)^\Omega : x \text{ is } \mathscr{F}_t\text{-measurable}\}$ for all $t \in T$. An *assignment* in $\mathscr{E}$ is a function $f : T \times \Omega \to Y_+$ such that $f(\cdot, \omega) \in L_1(\mu, Y)$ for all $\omega \in \Omega$, and an assignment f is called an *allocation* if $f(t, \cdot) \in L_t$ for almost all $t \in T$. Following Hervés-Beloso and Moreno-García (2008), we assume free disposal. An assignment f in $\mathscr{E}$ is *S-feasible* if $\int_S f(\cdot, \omega) d\mu \leq \int_S a(\cdot, \omega) d\mu$ for all $\omega \in \Omega$. For simplicity, a T-feasible assignment is termed as feasible assignment. Throughout the paper, we pose the following standard assumption on the initial endowments.

(A_1) $a(\cdot, \omega) \in L_1(\mu, Y)$ for all $\omega \in \Omega$, $a(t, \cdot) \in L_t$ for almost all $t \in T$, and $a(t, \omega) \gg 0$ for all $(t, \omega) \in T \times \Omega$.

This assumption has been used in many references; see Evren and Hüsseinov (2008) and Hervés-Beloso et al. (2005b). A *price system* is a non-zero function $\pi : \Omega \to Y^*_+$, where Y^*_+ is the positive cone of the norm-dual space Y^* of Y. The *budget set* of agent t with respect to a price system π is defined by

$$B_t(\pi) = \left\{ x \in L_t : \sum_{\omega \in \Omega} \langle \pi(\omega), x(\omega) \rangle \leq \sum_{\omega \in \Omega} \langle \pi(\omega), a(t, \omega) \rangle \right\}.$$

A *Walrasian expectations equilibrium* of $\mathscr{E}$ in the sense of Radner is a pair (f, π), where f is a feasible allocation and π is a price system such that for almost all $t \in T$, $f(t, \cdot) \in B_t(\pi)$, $f(t, \cdot)$ maximizes V_t on $B_t(\pi)$ and

$$\sum_{\omega \in \Omega} \left\langle \pi(\omega), \int_T f(\cdot, \omega) d\mu \right\rangle = \sum_{\omega \in \Omega} \left\langle \pi(\omega), \int_T a(\cdot, \omega) d\mu \right\rangle.$$

In this case, f is called a *Walrasian expectations allocation* and the set of all such allocations is denoted by $\mathscr{W}(\mathscr{E})$.

The following monotonicity assumption has been used in many references, for instance, Evren and Hüsseinov (2008) and Hervés-Beloso et al. (2005b).

(A_2) For each $(t, \omega) \in T \times \Omega$, $U_t(\omega, \cdot) : Y_+ \to \mathbb{R}$ is strictly monotone, that is, $U_t(\omega, x + y) > U_t(\omega, x)$ if $x, y \in Y_+$ with $y \gg 0$.

Let $\mathfrak{P}$ denote the set of partitions on Ω. For a $\mathscr{Q} \in \mathfrak{P}$, $T_{\mathscr{Q}} = \{t \in T : \Pi_t = \mathscr{Q}\}$. We assume that $T_{\mathscr{Q}} \in \Sigma$ for all $\mathscr{Q} \in \mathfrak{P}$. For any coalition S, Σ_S and μ_S denote the restrictions of Σ and μ on S, and $\mathfrak{P}(S) = \{\mathscr{Q} \in \mathfrak{P} : \mu(S \cap T_{\mathscr{Q}}) > 0\}$.

2.2. An interpretation via an atomless economy

A classical result in measure theory asserts that T can be decomposed into two parts: one part is atomless and the other is the union of at most countably many pairwise disjoint atoms. That is, $T = T_0 \cup T_1$, where T_0 is the atomless part and T_1 is the union of at most countably many μ-atoms. Since each μ-atom is treated as an agent, if A is a μ-atom, then $A \in T_1$ is used, instead of writing $A \subseteq T_1$. Agents in T_0 are called *small agents* and those in T_1 are called *large agents*. Let $\mathscr{A} = \{A_i : i \geq 1\}$ be the collection of atoms in T.

Following Greenberg and Shitovitz (1986), we associate $\mathscr{E}$ with an atomless economy $\mathscr{E}^*$. The space of agents of $\mathscr{E}^*$ is denoted by (T^*, Σ^*, μ^*), where $T^* = T_0 \cup T^*_1$ and T^*_1 is an atomless measure space such that $\mu^*(T^*_1) = \mu(T_1)$ and $T_0 \cap T^*_1 = \emptyset$. We assume