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Parimutuel betting under asymmetric information

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Abstract

This paper examines simple parimutuel betting games under asymmetric information, with particular attention to differences between markets in which bets are submitted simultaneously versus sequentially. In the simultaneous parimutuel betting market, all (symmetric and asymmetric) Bayesian–Nash equilibria are generically characterized as a function of the number of bettors and the quality of their private information. There always exists a separating equilibrium, in which all bettors follow their private signals. This equilibrium is unique if the number of bettors is sufficiently large. In the sequential framework, earlier bets have information externalities, because they may reveal private information of bettors. They also have payoff externalities, because they affect the betting odds. One effect of these externalities is that the separating equilibrium disappears if the number of betting periods is sufficiently large.

JEL classification: C72; D82

Keywords: Parimutuel betting; Asymmetric information; Information aggregation; Herd behavior; Contrarian behavior

1. Introduction

In 2002, Deutsche Bank and Goldman Sachs introduced options on economic data releases such as employment, retail sales, industrial production, inflation, and economic growth, with the purpose of providing a means of hedging core risk. These new economic derivatives are priced and allocated *parimutuelly*, meaning that their prices are based solely on the relative demand for their implied outcomes. The parimutuel system is also the accepted betting procedure at major horse-racing tracks throughout the world, where investments on the winning horse yield returns that are decreasing with the proportion of bettors who have bet on the same horse.

Several empirical and theoretical studies have investigated the properties of parimutuel betting markets and have pointed out their relevance for the analysis of decision making under risk and market efficiency.[1] Empirical research establishes that, although there is a tendency in parimutuel betting markets for price changes to move in the direction

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[1] A detailed account of this literature may be found in Sauer (1998) and Vaughan Williams (1999).

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of actual outcomes over time, some empirical regularities are clearly inconsistent with informational efficiency. The most robust anomalous empirical regularity, called the *favorite-longshot bias*, is characterized by the win pool shares being lower than win frequencies for favorites, and higher than win frequencies for longshots. It is thus generally more profitable to bet on favorites than on longshots.

Most of the existing theoretical studies of parimutuel betting have focused on games with symmetric information, either by modeling bettors as having homogeneous beliefs (Chadha and Quandt, 1996; Feeney and King, 2001), including uninformed bettors modeled as noise bettors (Hurley and McDonough, 1995; Terrell and Farmer, 1996; Koessler et al., 2003), or endowing all bettors with inconsistent beliefs (Watanabe et al., 1994; Watanabe, 1997). In contrast to previous studies, the model of parimutuel betting introduced here considers an environment in which differences in beliefs are due only to differences in privately held information about the probability of each outcome. For the example of a horse race, such differences in information may be due, e.g., to the dispersion of knowledge concerning the intrinsic ability of each horse, the condition of the track, the skill of each jockey, the horses' performances in previous races, etc. ... Our focus is on the properties of equilibria of betting games in this environment and their relationship to the underlying informational structure.

The most closely related theoretical analysis of parimutuel betting is a recent model of Ottaviani and Sørensen (2005). They propose two models of parimutuel betting with, as in our model, risk-neutral players, two states of nature, and a prior probability distribution over states that is common for all bettors. In the first of their two models, there is a finite number of symmetrically informed bettors who choose the time and the exact amount of money they want to bet on an outcome. In their second model, there is a continuum of privately informed bettors who bet an indivisible unit of money simultaneously. Ottaviani and Sørensen (2005) show, among other results, that both types of models can generate a favorite-longshot bias.

In this paper, contrary to Ottaviani and Sørensen (2005), we model the timing of bets as exogenous and analyze the difference between simultaneous and sequential betting. We first consider the simultaneous move version of the game. Generically (for almost all bettors' qualities of information), we characterize the set of all (symmetric and asymmetric) pure strategy Bayesian–Nash equilibria as a function of the number of bettors and the quality of their private information. In particular, we show that a separating equilibrium, in which all bettors always bet consistently with their private information, always exists. Asymmetric equilibria, in which some players bet on the same horse whatever their private signal, also exist. Although such a multiplicity of equilibria raises selection questions, we show that the selection issue is irrelevant with a large number of bettors, since asymmetric equilibria vanish when the number of bettors increases. Consequently, in large, simultaneous wagering markets, the odds against each horse always reflect all private information. However, as in Ottaviani and Sørensen's (2005) second model, a favorite-longshot bias is obtained because market odds, which reflect individuals' average beliefs, are less extreme than the beliefs that would have been obtained from the aggregation of all private signals.

Next, we investigate a market in which bets are submitted sequentially. The sequential version takes into account information externalities, resulting from the inferences later bettors can draw from the decisions of earlier bettors, and the related strategic incentives to influence later bettors. It also includes payoff externalities resulting from earlier bettors' influence on the odds, and thus the payoffs to each action, that later bettors face. We show that a separating equilibrium can fail to exist, and will typically disappear when the number of bettors increases. This non-existence is related to the fact that bettors exhibit *herd behavior*, as in standard models of information cascades (Banerjee, 1992; Bikhchandani et al., 1992), which is the imitation of predecessors' choices while overriding own private information. However, as in most multi-agent sequential trade models with asymmetric information (e.g., Avery and Zemsky, 1998), the price mechanism ensures that complete imitation does not continue indefinitely.

The paper proceeds as follows: Section 2 lays out the basic model of parimutuel betting used throughout the paper. In Section 3, we present the results concerning the simultaneous move betting game. Section 4 investigates the sequential move version of the game. We relate our study to earlier work on social learning in multi-person sequential decision problems in Section 5. The appendix contains the proofs for some of the results of Sections 3 and 4.

2. A model of parimutuel betting

We consider an environment with two possible outcomes, which for ease of exposition can be thought of as a horse race where there are only two horses. Each member of a group of bettors is required to bet one unit of money on