Games and Economic Behavior 69 (2010) 42–71

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Price dynamics on a stock market with asymmetric information

Bernard De Meyer [1]

PSE-CES, Université Paris 1, 112 Bd de l'Hôpital, 75013 Paris, France

ARTICLE INFO

Article history:
Received 19 February 2007
Available online 6 February 2010

JEL classification:
G14
C72
C73
D44

Keywords:
Repeated games
Incomplete information
Price dynamics
Martingales of maximal variation

ABSTRACT

When two asymmetrically informed risk-neutral agents repeatedly exchange a risky asset for numéraire, they are essentially playing an n-times repeated zero-sum game of incomplete information. In this setting, the price L_q at period q can be defined as the expected liquidation value of the risky asset given players' past moves. This paper indicates that the asymptotics of this price process at equilibrium, as n goes to ∞, is completely independent of the "natural" trading mechanism used at each round: it converges, as n increases, to a Continuous Martingale of Maximal Variation. This martingale class thus provides natural dynamics that could be used in financial econometrics. It contains in particular Black and Scholes' dynamics. We also prove here a mathematical theorem on the asymptotics of martingales of maximal M-variation, extending Mertens and Zamir's paper on the maximal L^1-variation of a bounded martingale.

1. Introduction

One fundamental problem in financial analysis is to accurately identify the stock price dynamics: different dynamics for the underlying asset will lead to different pricing formulae for derivatives.

Financial econometrics does not completely solve this problem: Statistical methods can calibrate the parameters of a model, finding in a general class of possible dynamics, the one that best fits the historical data. But still, assumptions have to be made regarding the class of possible dynamics. Most of the classes used in practice (Bachelier's dynamics, Black and Scholes dynamics, diffusion models, stochastic volatility models, GARCH-models, etc.) are chosen by a kind of rule of thumb, with no real economic justification. The randomness of the prices is often conceived as completely exogenous. The first sentences in Bachelier's (1900) thesis illustrate quite well this kind of explanation: "The influences that determine the price variations on the stock market are uncountable. Past, present or even future expected events, having often nothing to do with the stock market, have repercussion on the prices."

In this paper however, we suggest that part of the randomness in the stock price dynamics is endogenous: it is introduced by the agents in order to maximize their profit. This idea was already present in De Meyer and Moussa-Saley (2003), where the Brownian term in the price dynamics was explained endogenously. Institutional investors clearly have better access to information on the market than the private ones: they are better skilled to analyze the flow of information and in some cases they are even part of the board of directors of the firms of which they are trading the shares. So, institutional investors are better informed and this informational advantage is known publicly. As a consequence, each of their moves on the markets is analyzed by the other agents to extract its informational content. If informed agents act naively, making

E-mail address: demeyer@univ-paris1.fr.

[1] The main ideas of this paper were developed during my stay at the Cowles Foundation for research in Economics at Yale University. I am very thankfull to all the members of the Cowles foundation for their hospitality. I would like to thank John Geanakoplos and Pradeep Dubey for fruitful discussions.

0899-8256/$ – see front matter
doi:10.1016/j.geb.2010.01.011

moves that depend deterministically on their information, they will completely reveal this information to the other agents, and doing so, they will lose their strategic advantage for the future. The only way to benefit from the information without revealing it too fast is to introduce noise on their moves: this is tantamount to selecting random moves via lotteries that depend on their information. The main idea in De Meyer and Moussa-Saley (2003) is that the noise introduced by the informed agents in the day-to-day transactions will generate a Brownian motion.

The central result of this paper is that this kind of argument leads to a particular class of price dynamics, hereafter referred to as Continuous Martingales of Maximal Variation (CMMV), which is quite robust: CMMV will appear in any repeated exchange between two risk neutral asymmetrically-informed players, independently of the "natural" trading mechanism used in the exchanges.

The structure of the paper is as follows: in the next section, we define CMMV precisely. As suggested by our main result, this class of dynamics is a natural candidate that could be used in financial econometrics as the class of possible dynamics. As will be seen, this class of dynamics is a subclass of local volatility models that contains as particular cases Bachelier's dynamics as well as Black and Scholes' dynamics.

In Section 3, we introduce the game Γ_n to model the repeated exchanges between two risk neutral asymmetrically informed players in the most general way: Player 1 initially receives a private message concerning the liquidation value L of the risky asset traded. During n consecutive rounds, the players exchange the risky asset against a numéraire, using a general trading mechanism $\langle I, J, T\rangle$ which is simply a game with respective action spaces I and J for players 1 and 2, and whose outcome $T(i, j)$ is a transfer vector representing the quantities of risky asset and numéraire exchanged when actions are (i, j). At each round, actions are chosen simultaneously and are then publicly announced. The players aim to maximize the liquidation value of their final portfolio.

The game Γ_n is equivalent to a zero sum repeated game with one sided information à la Aumann–Maschler. In Section 4, we define the concepts of strategy, value and optimal strategy for Γ_n. Since players are risk neutral, the natural notion of price L_q of the risky asset at period q is defined as the conditional expected liquidation value given player 2's previous observations.

The trading mechanism introduced above should satisfy some properties in order to represent real exchanges on the stock market. In Section 5, we introduce 5 axioms that must be satisfied by a "natural" trading mechanism. Let us describe them very briefly here:

(H1) The game Γ_n has a value V_n whatever the distribution of the liquidation value is.
(H2) is a continuity assumption of V_1 as a function of the law of the liquidation value.
(H3) stipulates that the mechanism should be invariant with respect to the scale of numéraire: If two players use this mechanism to exchange a risky asset R against the dollar or against the cent, the same transactions in value will be observed in both cases. More specifically, the quantity of risky asset exchanged will be the same, but the counterpart in cents will be the counterpart in dollars multiplied by 100. (H3) is thus a 1-homogeneity property of the value.
(H4) is an invariance axiom with respect to the riskless part of the risky asset: If two players use the trading mechanism to exchange with the dollar as numéraire an asset R' consisting of one share of asset R and a \$100 bill, then the transaction observed will be the same in value as if they were exchanging R for the dollar. In other words, the quantity x of R- and R'-shares exchanged will be the same in both cases, but the x bills of \$100 exchanged within the R'-shares will be paid back in dollars, that is, if y and y' denotes the counterpart in numéraire when exchanging R and R' respectively, then $y' = y + 100x$. The value of the game must thus remain unchanged if one shifts the liquidation value by a constant amount.
(H5) There exists a situation in which player 1 can take a strictly positive profit from his private information: he is strictly better off with his message than without. This axiom is on the one hand completely natural to model the stock market: it seems indeed commonsense that private information has a strictly positive value on the market. Otherwise, there would clearly be no need for insider trading regulation, since no one would have incentive to make such trades.

On the other hand, however, this axiom is in a way unnatural. This game is zero sum and has a positive value. So why should the uninformed player participate in a game where he is loosing money? This is a particular case of Milgrom–Stokey's No Trade Theorem. Some agents on the market are in fact forced to trade: for instance, a market maker facing a more informed trader. Since the bid and the ask posted by a market maker is a commitment to buy or sell at these prices any quantity of shares up to a prefixed limit, the only way for the market maker to avoid trading would be to post a very large bid-ask spread. Most market regulations however impose explicit limit on market makers' bid-ask spread, thus steering past the No Trade paradox.

At the end of Section 5, we state the main result of the paper which is Theorem 1. It indicates that if the trading mechanism is natural in the above sense, if the price process $(L_q)_{q=0,\dots,n}$ at equilibrium in Γ_n is represented by the continuous time process $(\Pi^n_t)_{t\in[0,1]}$, with $\Pi^n_t := L_q$ on the time interval $[q/n, (q+1)/n[$, then Π^n converges in law to a particular CMMV Π^μ depending just on the law μ of the liquidation value of R. The limit is thus completely independent of the natural trading mechanism considered, showing in this way the robustness of the CMMV class of dynamics. This paper differs from the existing literature on trading with asymmetrical information (see e.g. Kyle, 1985) by the fact that the price randomness is essentially considered as endogenous. In Kyle's paper however, to get rid of the above mentioned No Trade