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Blocking efficiency in an economy with asymmetric information

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ABSTRACT

In this paper, we study a pure exchange atomless economy with asymmetric information and having an ordered Banach space with an interior point in its positive cone as the commodity space. An extension of the main theorem in Vind (1972) to the private core without free disposal is established. As a particular case of this result, a solution to a problem mentioned in Pesce (2010) is derived.

1. Introduction

One of the disadvantages of the classical Arrow–Debreu–McKenzie model is that the influence of each individual agent is non-negligible, thus the competition is imperfect. To achieve perfect competition, Aumann (1964) considered an economic model with a continuum of small agents (i.e., an atomless economy), and proved the equivalence of the core and the set of Walrasian allocations for finitely many commodities. Eight years later, three notes in the same issue of *Econometrica* gave a sharper interpretation to Aumann's core-Walras equivalence theorem as a characterization of perfect competition. Firstly, Schmeidler (1972) proved that if an allocation f is blocked by a coalition S via an allocation g, then for any $\varepsilon > 0$, f can be blocked via the same allocation g by a coalition $S' \subseteq S$ with $\mu(S') \leq \varepsilon$. Schmeidler's result was further generalized in Grodal (1972) by restricting the set of coalitions to those consisting of finitely many arbitrarily small sets of agents with similar characteristics, which are presumably easier to form and also interpret. Precisely, Grodal proved that an allocation belongs to the core if and only if it cannot be blocked by a coalition which is the union of at most $\ell + 1$ sub-coalitions, each of which has measure and diameter less than ε, where ℓ denotes the number of commodities. Finally, Vind (1972) showed that if some coalition blocks a feasible allocation then there is a blocking coalition with any measure less than the measure of the grand coalition. These results imply that, for a finite-dimensional commodity space, the set of Walrasian allocations of an atomless economy coincides with the set of allocations that are not blocked by coalitions of arbitrarily given measure less than that of the grand coalition. It is well known that similar results do not hold if one restricts the economy to finitely many agents. Besides this impossibility, Khan (1974) showed that restrictions placed on the formation of coalitions as in Schmeidler (1972), Grodal (1972) and Vind (1972) do not enlarge the core "very much" if the number of agents in the finite economy is large enough.

Since the publication of Aumann (1964), the relation between the core and the set of Walrasian allocations has been studied in the context of infinite dimensional commodity spaces by many authors. In this context, the relation between the two sets of allocations is more interesting, since preferences and endowments are more diverse, and thus blocking becomes more difficult; refer to Hervés-Beloso et al. (2000). Thus, it is interesting to examine whether the results of Schmeidler (1972), Grodal (1972) and Vind (1972) are extendable to infinite dimensional commodity spaces. Since these results rely heavily on Lyapunov's convexity theorem, which does not hold in an infinite dimensional setting, the exact extension of either Schmeidler's or Grodal's result is not possible, as mentioned in Hervés-Beloso et al. (2000). Indeed, Núñez (1993) gave an example of an atomless economy, with infinitely many commodities, where an allocation f is blocked by the grand coalition via an allocation g, but there is no other different coalition blocking f via the same allocation g. Despite the impossibility for obtaining both of these results in their exact strong versions, Hervés-Beloso et al. (2000) showed that in a continuum economy whose commodity space is ℓ^∞, the space of bounded sequences, the following property holds: if an allocation f is blocked by a coalition via an allocation g, then there exist a

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natural number $N = N(f)$ and an allocation h such that for any $\varepsilon > 0$, f can be blocked via h by a subcoalition which is the union of at most N many sets, each of which has measure and diameter less than ε. Here, preferences are assumed to be Mackey continuous. Recently, Evren and Hüsseinov (2008) further extended the results of Schmeidler, Grodal and Vind to economies whose commodity spaces are ordered Banach spaces.

Another disadvantage of the Arrow–Debreu–McKenzie model is that it does not capture the uncertainty with asymmetric information. To overcome this shortcoming, Radner (1968) introduced an economic model consisting of finitely many agents, each of whom is characterized by a private information set, a state-dependent utility function, a random initial endowment and a prior belief. The trade of an agent is measurable with respect to his information so that he cannot act differently on states that he cannot distinguish and an agent makes a contract for trading commodities before he obtains any information about the realized state of nature. Radner also extended the notion of a Walrasian equilibrium in the Arrow–Debreu–McKenzie model to that of a Walrasian expectations equilibrium in his model so that better informed agents are generally better off. Since Radner's work, many authors have tried to extend the theory of deterministic economies to asymmetric information economies. For example, it was mentioned in the appendix of Evren and Hüsseinov (2008) that the extensions of Schmeidler's, Grodal's and Vind's theorems in Evren and Hüsseinov (2008) can be further extended to a framework with asymmetric information. On the other hand, several extensions of Vind's theorem to asymmetric information economies with the equal treatment setting were provided in Bhowmik and Cao (2012) and Hervés-Beloso et al. (2005a,b). Recently, using the notion of information sharing rule, Hervés-Beloso et al. (2011) established some results similar to those in Schmeidler (1972) and Vind (1972) in asymmetric information economies. Interestingly, the afore-mentioned results in Bhowmik and Cao (2012), Evren and Hüsseinov (2008) and Hervés-Beloso et al. (2005a,b, 2011) were obtained under the free disposal condition. When feasibility is defined with free disposal, Walrasian expectations equilibrium allocations may not be incentive compatible and contracts may not be enforceable, refer to Angeloni and Martins-da-Rocha (2009). Thus, to avoid this problem, it is desirable to consider a framework without free disposal. Recently, Angeloni and Martins-da-Rocha showed that Aumann's equivalence theorem is still valid in a framework with asymmetric information and without free disposal. As a result, whether Vind's theorem is still valid in the same framework emerges as a question. Indeed, as mentioned in Pesce (2010), whether there is a version of a Vind-type theorem on the private core of an asymmetric information economy without free disposal even for a finite dimensional commodity space is still an open problem.

The main purpose of this paper is to examine whether the results of Schmeidler (1972), Grodal (1972) and Vind (1972) are extendable to the context of pure exchange atomless and asymmetric information economies without free disposal and having infinitely many commodities. The rest of this paper is organized as follows. In Section 2, we give a general description on our model. In Section 3, we provide an infinite dimensional extension of Vind's theorem in our context, including its full proof and some applications. For particular interests, we also provide a variation of Grodal's theorem in our framework. In Section 4, we continue our investigation and then obtain Vind-type theorems for other core solutions, namely, the fine core and the coarse core, introduced in Wilson (1978) and Yannelis (1991a). Throughout the paper, we do not assume the free disposal constraint.

2. The model

We consider a pure exchange economy $\mathscr{E}$ with asymmetric information. The space of states of nature is a probability space $(\Omega, \mathscr{F})$, where Ω is a finite set and the σ-algebra $\mathscr{F}$ on Ω denotes the family of all possible events. The space of agents is a measure space (T, Σ, μ) with a complete, finite, positive and atomless measure μ, where T is the set of agents, Σ is the σ-algebra of measurable subsets of T whose economic weights on the market are given by μ. In each state, infinitely many commodities are assumed. Throughout the paper, the commodity space of $\mathscr{E}$ is an ordered Banach space Y whose positive cone has an interior point. The order on Y is denoted by $\leq$, and $Y_+ = \{x \in Y : x \geq 0\}$ denotes the positive cone of Y. The symbol $x \gg 0$ (resp. $x > 0$) means that x is an interior (resp. a non-zero) point of Y_+. Let $Y_{++} = \{x \in Y_+ : x \gg 0\}$. The economy extends over two time periods $\tau = 0, 1$. Consumption takes place at $\tau = 1$. At $\tau = 0$, there is uncertainty over the states and agents make contracts that are contingent on the realized state at $\tau = 1$. Thus, $\mathscr{E}$ can be defined by

$$\mathscr{E} = \{(\Omega, \mathscr{F}); (T, \Sigma, \mu); Y_+; (\mathscr{F}_t, U_t, a(t, \cdot), q_t)_{t \in T}\}.$$

Here, Y_+ is the *consumption set* in every state $\omega \in \Omega$ for every agent $t \in T$; $\mathscr{F}_t$ is the σ-algebra generated by a partition Π_t of Ω representing the *private information* of agent t; $U_t : \Omega \times Y_+ \to \mathbb{R}$ is the *state-dependent utility function* of agent t; $a(t, \cdot) : \Omega \to Y_+$ is the *random initial endowment* of agent t, assumed to be constant on elements of Π_t; and q_t is a probability measure on Ω giving the *prior* of agent t. It is assumed that q_t is positive on all elements of Ω. The quadruple $(\mathscr{F}_t, U_t, a(t, \cdot), q_t)$ are called the *characteristics* of the agent $t \in T$. A function $x : \Omega \to Y_+$ is interpreted as a random consumption bundle in $\mathscr{E}$. The *ex ante expected utility* of an agent t for a given random consumption bundle x is defined by $V_t(x) = \sum_{\omega \in \Omega} U_t(\omega, x(\omega)) q_t(\omega)$.

Let $L_1(\mu, Y)$ denote the set of all Bochner integrable functions from T into Y, and $L_t = \{x \in (Y_+)^{\Omega} : x \text{ is } \mathscr{F}_t\text{-measurable}\}$. Throughout the paper, we pose the following assumption on the initial endowments.

(A_1) $a(\cdot, \omega) \in L_1(\mu, Y)$ for all $\omega \in \Omega$, and $a(t, \omega) \gg 0$ for all $(t, \omega) \in T \times \Omega$.

This is a standard assumption and has been used in many references, see Evren and Hüsseinov (2008) and Hervés-Beloso et al. (2005a). Note that in the literature, some authors used a slightly weaker assumption: $\int a(\cdot, \omega) d\mu \gg 0$ for all $\omega \in \Omega$, for instance, Angeloni and Martins-da-Rocha (2009) and Hervés-Beloso and Moreno-García (2008). For any $n \geq 1$, the $(n-1)$-simplex of $\mathbb{R}^n$ is defined as

$$\Delta^n = \left\{ x = (x_1, \ldots, x_n) \in \mathbb{R}^n_+ : \sum_{i=1}^{n} x_i = 1 \right\}.$$

Define a function $\varphi : (T, \Sigma, \mu) \to \Delta^{|\Omega|}$ by $\varphi(t) = q_t$ for all $t \in T$. For each $\omega \in \Omega$, define a function $\psi_\omega : T \times Y_+ \to \mathbb{R}$ by $\psi_\omega(t, x) = U_t(\omega, x)$. We shall need the following assumption on the priors of agents.

(A_2) The function φ is measurable, where $\Delta^{|\Omega|}$ is endowed with the Borel structure.

Furthermore, the following two assumptions on the utility functions will be needed.

(A_3) For each $\omega \in \Omega$, the function ψ_ω is Carathéodory, that is, $\psi_\omega(\cdot, x)$ is measurable for all $x \in Y_+$, and $\psi_\omega(t, \cdot)$ is norm-continuous for all $t \in T$.

(A_4) For each $(t, \omega) \in T \times \Omega$, $U_t(\omega, \cdot) : Y_+ \to \mathbb{R}$ is strictly monotone, that is, $U_t(\omega, x + y) > U_t(\omega, x)$ if $x, y \in Y_+$ with $y \gg 0$.

Assumptions (A_2) and (A_3) were used in Evren and Hüsseinov (2008). Assumption (A_4) is standard, and was used in Evren and Hüsseinov (2008) and Hervés-Beloso et al. (2005b). Under (A_2) and