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Equilibrium theory with asymmetric information and with infinitely many commodities

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Abstract

The traditional deterministic general equilibrium theory with infinitely many commodities cannot cover economies with private information constraints on the consumption sets. We bring the level of asymmetric information equilibrium theory at par with that of the deterministic one. In particular, we establish results on equilibrium existence for exchange economies with asymmetric (differential) information and with an infinite dimensional commodity space. Our new equilibrium existence theorems include, as a special case, classical results, e.g. Bewley [Existence of equilibria in economies with infinitely many commodities, J. Econ. Theory 4 (1972) 514–540] or Mas-Colell [The price equilibrium existence problem in topological vector lattices, Econometrica 54 (1986) 1039–1053].

JEL classification: D00; D50; D60; D82; D86

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1. Introduction

Uncertainty was introduced in equilibrium theory by Arrow and Debreu. Both authors realized (see, for example, Chapter 7 of the classical treatise of Debreu [9]) that if the exogenous uncertainty is described by a set which denotes the states of nature of the world and agents' characteristics, i.e. preferences and initial endowments, become random (state dependent), then the classical equilibrium results on existence and optimality of the Walrasian equilibrium continue to hold.

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This is the so-called "state contingent" model which captures the meaning of contracts (or trades) under uncertainty. However, this model does not allow for trades to be made under asymmetric information as agents' uncertainty is common. In a seminal paper, Radner [22] allowed, in addition to the random preferences and initial endowments, each agent to have her own private information set which was described by a partition of an exogenously given set of states of nature. In this model optimal choices reflect the private informations of the agents as net trades of an agent are measurable with respect to her private information partition, i.e. measurable with respect to the σ-algebra that her information partition generates. Furthermore, the market clearing occurs in the sense that total consumption is equal (or less than equal, which amounts to free disposal) to the total initial endowment for each state of nature. Thus, Radner [22] introduced asymmetric information in the Arrow–Debreu model.

Concerning the assumption of free disposal, Radner himself realized that this assumption may be problematic in the context of asymmetric information. [1] Indeed, as it was shown in Glycopantis et al. [13], the free disposal assumption may destroy the incentive compatibility of the Walrasian equilibrium and thus the resulting trades (contracts) need not be incentive compatible. Also, the free disposal assumption results in allocations which are not consistent with Bayesian rationality, i.e. they may not be implementable as a perfect Bayesian equilibrium of an extensive form game. On the other hand, it is easy to construct examples of well-behaved differential information economies where no Walrasian equilibrium exists with positive prices and without free disposal (see [13]).

The main purpose of this paper is to extend the theory of differential information economies to infinite dimensional commodity spaces. In particular, we prove the existence of a Walrasian equilibrium for an economy with asymmetric (differential) information. The commodity spaces treated are general enough to include most infinite dimensional spaces appearing in equilibrium theory. Moreover, we allow for very general preferences, i.e. preferences need not be transitive or complete. Furthermore, our results are established without the assumption of free disposal.

Despite the fact that infinite dimensional commodity spaces have been introduced in order to capture the meaning of uncertainty, or commodity differentiation, or of an infinite time horizon, none of the existing models with an infinite dimensional commodity space allow for asymmetric information and no free disposal simultaneously. Our results indicate that such a generalization is possible and thus we bring the level of asymmetric information equilibrium theory at par with that of the deterministic one.

However, there are several technical difficulties which need to be bypassed. In fact, the infinite dimensional standard arguments and results are not directly applicable. The reason is that the (informationally constrained) consumption sets do not coincide with the positive cone of the commodity space, and also are not upper comprehensive; rather the (informationally constrained) consumption sets are located in "thin" subspaces of the commodity space. In particular, even when the positive cone of the commodity space has non-empty interior, this is not so for the consumption sets. Moreover, for the case where the positive cone of the commodity space has an empty interior, properness assumptions on preferences and techniques like the Riesz–Kantorovich formula (see e.g. [2]) do not work immediately when trying to follow the standard approach of obtaining an equilibrium for an economy from an equilibrium relative to the restriction of the economy to the order ideal generated by the aggregate endowment. A particular problem here is that price

[1] To quote [22] on this point: "…the assumption of free disposal is inappropriate here, at least in its usual form; even though a given act α is compatible with an information structure $\mathcal{S}$, the set of acts $\leqslant \alpha$ (in the vector sense) will typically contain acts that are not compatible with $\mathcal{S}$."