



# An economic order quantity model with multiple partial prepayments and partial backordering

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## ABSTRACT

The classic Economic Order Quantity model assumes that an order is paid for at the time of its receipt. In practice, suppliers may require purchasers to pay a fraction of the order's cost in advance and sometimes allow them to divide the prepayment into multiple equal-sized parts to be paid during a fixed lead time. In this paper, an EOQ model with multiple prepayments under three different conditions: (a) no shortage, (b) full backordering and (c) partial backordering, are developed. Numerical examples illustrate the proposed models and solution method.

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## 1. Introduction and literature review

Two factors that can have a significant impact on the optimal order quantity are the timing of the payment for the item ordered and customers' reactions when the vendor runs out of stock.

When an order is placed with a supplier, there are three basic possibilities with respect to when that order is paid for: (1) at the time the order is placed or at some other time prior to delivery (prepayment), (2) at the time of delivery, or (3) at some time after delivery (delayed payment). In some cases the timing of payments may be a combination of two or even all three of these possibilities, with part of the cost due in advance, more due at the time of delivery, and the rest due at a later date. Using the classic EOQ model as a framework, Bregman [1] analyzed the effects of the timing of disbursements on the order quantity, showing that large timing differences may significantly affect order quantities. In a comment on Bregman's paper, Lau and Lau [2] found that, although the effect on the order quantity may be large, the effect on the overall cost may be relatively small.

The basic EOQ model developed by Harris [3] implicitly assumes that the customer pays for the items when they are received. This assumption also applies to the standard basic EOQ models with full or partial backordering and is used in most of the inventory literature dealing with extensions of those basic models.

Goyal [4] was the first to develop an EOQ model in which the purchasing cost could be paid some time after receiving products, a practice followed extensively in industrial and commercial purchasing. Relaxation of the EOQ model with delayed payment's assumption that stockouts are not permitted led to the development of a full-backorder model for deteriorating inventory by Jamal et al. [5]. Chang and Dye [6] developed a model to determine the optimal ordering policy for deteriorating items when delayed payments and partial backordering are assumed.

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As discussed in [7], suppliers sometimes require purchasers to pay a fraction of the purchasing cost as prepayment and may allow them to make this prepayment in installments. There are situations in which the purchaser may get some price discount at the time of final payment if he makes an extra advance payment. Maiti et al. [8] studied the effect of advanced payment on the total profit and inventory decisions when the carrying, ordering, purchasing and advertising costs are constants. Gupta et al. [7], in addition to including partial backordering in the model, treated the cost parameters as interval, rather than fixed, values, using a Genetic Algorithm to determine an optimal solution. Taleizadeh et al. [9] developed a multi-product, multi-constraint inventory control problem to import raw material from another country for which a fraction of purchasing cost should be paid as prepayment. Sharma and Sadiwala [10] modeled a composite inventory system that included partial backordering and investigated the sensitivity of the order and shortage quantities and total cost of the inventory control system to changes in the backordering rate. Jolai et al. [11] developed a production model for deteriorating items in which partial backordering, stock-dependent demand and inflation issues are considered.

The first model that made all the usual assumptions of the classic EOQ model with the addition of partial backordering of demand during a stockout period was developed by Montgomery et al. [12], with several other authors since developing comparable models. A survey by Pentico and Drake [13] describes them, along with other deterministic models that include not only partial backordering but a variety of other considerations, such as a time-dependent backordering rate, deteriorating inventory and time, price, or inventory-level-dependent demand patterns. The partial backordering model that is the basis for the model we present here was developed by Pentico and Drake [14].

In this paper we will develop models for the optimal order quantity, changing only two of the basic assumptions of the classic economic order quantity (EOQ) model. First, we assume that the supplier requires multiple partial prepayments between the time the order is placed and the time of receipt. Second, we consider three possibilities with respect to stockouts: no stockouts allowed (the classic EOQ model's assumption), full backordering of stockouts, and partial backordering of stockouts.

## 2. Problem definition

Consider a situation in which a vendor requires the purchaser to prepay a fraction  $\alpha$  of the purchasing cost before the order is received, with the first prepayment being at a specified time  $L$  before delivery. The vendor may require that the prepayment be made in a single lump sum; in other cases it can be paid in multiple installments at equal intervals. In either of these situations, when the buyer receives the order he has already paid  $\alpha$  percent of the total purchasing cost, with the balance due on delivery. In this purchasing system the capital cost for the buyer will increase because he has incurred interest costs on  $\alpha$  percent of the purchasing cost of items that have not yet been received. Depending on the supplier's policy, the purchaser may have the option of offering how many prepayments to make during the fixed period of length  $L$ , but if the vendor asks the buyer to make a maximum of  $n$  prepayments, then that is the number he should make because his capital cost during the lead time will be minimized. So we consider  $n$ , the number of prepayments, to be a parameter of the problem. In the following, we develop economic order quantity models with multiple prepayments for three cases: no backordering, full backordering, and partial backordering.

## 3. Model development

Our models for all three of the cases we consider – no shortages, full backordering, and partial backordering – will be based on the structure of the EOQ with partial backordering model in [14], in which the decision variables were  $T$ , the cycle time or the time between orders, and  $F$ , the fill rate or the percent of demand filled from stock.

We introduce the parameters and variables of the model in Section 3.1 and in Section 3.2 our models are developed.

### 3.1. Notation

The following notation is used to model the problem.

#### Parameters:

$A$	Fixed order cost
$\alpha$	The fraction of purchasing cost that must be paid as multiple prepayments
$\beta$	The fraction of shortages that will be backordered
$C$	Purchase cost of an item
$D$	Demand per period
$g$	Goodwill loss for a unit of lost sales
$h$	Holding cost per unit per period including capital cost $h = (I_c + I_h) C$
$h_c$	Capital cost per unit per period $h_c = I_c C$
$I_c$	Capital cost rate per unit time
$I_h$	Holding cost rate per unit time excluding capital cost
$L$	The length of time during which the buyer will make the prepayments
$n$	The number of equally spaced prepayments to be made before receiving the order

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