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Fuzzy profit measures for a fuzzy economic order quantity model

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Abstract

Changing economic conditions make the selling price and demand quantity more and more uncertain in the market. The conventional inventory models determine the selling price and order quantity for a retailer's maximal profit with exactly known parameters. This paper develops a solution method to derive the fuzzy profit of the inventory model when the demand quantity and unit cost are fuzzy numbers. Since the parameters contained in the inventory model are fuzzy, the profit value calculated from the model should be fuzzy as well. Based on the extension principle, the fuzzy inventory problem is transformed into a pair of two-level mathematical programs to derive the upper bound and lower bound of the fuzzy profit at possibility level α. According to the duality theorem of geometric programming, the pair of two-level mathematical programs is transformed into a pair of conventional geometric programs to solve. By enumerating different α values, the upper bound and lower bound of the fuzzy profit are collected to approximate the membership function. Since the profit of the inventory problem is expressed by the membership function rather than by a crisp value, more information is provided for making decisions.

Keywords: Inventory; Geometric programming; Optimization; Fuzzy profit; Decision making

1. Introduction

The integration of production and marketing functions has been recognized to be crucial in practice for increasing a firm's profit and decreasing their conflicts by reducing losses incurred from separate decision making. Pricing and lot sizing are two important strategies that concern simultaneous determination of an item's price and lot size or economic order quantity (EOQ) to maximize a firm's profit for constant but price-dependent demands over a planning horizon. Different from the classic lot sizing or EOQ problem, the demand is typically determined as a decreasing power function of the selling price with constant elasticity as in monopolistic pricing situations [1]. Geometric programming is an efficient and effective method to solve nonlinear programming problem with the terms in power functional form in the objective function and constraints

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[2,3]. The applications of geometric programming techniques on inventory management are well documented in the literature. Worrall and Hall [4] utilize geometric programming technique to solve an inventory model with multiple items subject to multiple constraints. Cheng [5] proposed an EOQ model with demand-independent unit cost and derived the optimal solution by employing geometric programming technique. Lee [6] utilized geometric programming techniques to determine the selling price and order quantity for a retailer. Jung and Klein [7] employed geometric programming technique to analyze two EOQ based inventory models under total cost minimization and profit maximization. Chen [8] proposed an inventory model under return-on-inventory-investment maximization for an intermediate firm to determine the selling quantity and purchase cost of a product via geometric programming technique.

The parameters in those studies are assumed to be precisely known. However, in real world applications, the parameters in the model might be inexact and imprecise in nature. Today the selling price, purchase cost, and demand quantity of a technology product become more and more uncertain in the market. To cope with quantitatively with uncertain information in making decision, Zadeh [9] introduces the notion of fuzziness. The associated problem becomes a fuzzy inventory model. There are few studies discussing about the fuzzy profit measures in fuzzy economic order quantity model. Roy and Maiti [10] employ both fuzzy nonlinear and geometric programming techniques to solve a fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. Nevertheless, only crisp solutions are provided. Intuitively, when the parameters are fuzzy, the objective value should be fuzzy as well. Yao and Lin [11] derived the optimal fuzzy profit using the fuzzy price function and linear cost function. The results are identical when the fuzzy case was reduced to the crisp case. Toyonaga et al. [12] proposed a model of crop planning with fuzzy profit coefficients and extended this fuzzy model by setting the profit coefficients as discrete randomized fuzzy numbers. Similar to the study of Roy and Maiti [10], Yao and Lin [11] and Toyonaga et al. [12] provided only crisp solutions.

In this paper, we develop a solution procedure that is able to calculate the fuzzy profit of the fuzzy inventory model where the demand quantity and unit cost are represented as fuzzy numbers. The idea is to apply Zadeh's extension principle [13,9,14]. A pair of two-level mathematical programs is formulated to calculate the upper bound and lower bound of the profit at possibility level α. The membership function of the fuzzy profit is derived numerically by enumerating different values of α. The rest of this paper is organized as follow. We first state the problem and model formulation with fuzzy parameters. Next, a pair of two-level mathematical programs for calculating the upper bound and lower bound of the profit is formulated; we transform the two-level mathematical programs into one-level conventional geometric programs to solve. Then we use an example to illustrate the idea of this paper. Finally, some conclusions are drawn from the discussion.

2. Basic concepts on fuzzy sets

Fuzzy sets are generalizations of crisp sets as a way of representing imprecise or vagueness in real world. A fuzzy set is a collection of elements in a universe of information where the boundary of the set contained in the universe is ambiguous, vague and otherwise fuzzy. Each fuzzy set is specified by a membership function, which assigns to each element in the universe of discourse a value within the unit interval $[0,1]$. The assigned value called degree or grade of membership, which specifies the extent to which a given element belongs to the fuzzy set or is related to a concept. If the assigned value is 0, then the given element does not belong to the set. If the assigned value is 1, then the element totally belongs to the set. If the values lies within the interval $(0,1)$, then the element only partially belongs to the set. Therefore, any fuzzy set can be uniquely determined by its membership function.

Let X be the universe of discourse. A fuzzy set $\widetilde{A}$ of the universe of discourse X is said to be convex if and only if for all x_1 and x_2 in X there always exist [15,14]

$$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geqslant \mathrm{Min}(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)).$$

Fuzzy sets can also be represented by intervals, which are called α-level sets or α-cuts. The α-level sets A_α of a fuzzy set $\widetilde{A}$ are defined as

$$(A)_\alpha = \left[(A)_\alpha^{\mathrm{L}}, (A)_\alpha^{\mathrm{U}}\right] = [\min_x\{(x, \mu_{\widetilde{A}}(x)) | \mu_{\widetilde{A}}(x) \geqslant \alpha\}, \max_x\{(x, \mu_{\widetilde{A}}(x)) | \mu_{\widetilde{A}}(x) \geqslant \alpha\}].$$