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An analytic solution approach for the economic order quantity model with Weibull ameliorating items

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ABSTRACT

While deterioration models consider the gradual reduction in quality and quantity of products over time, amelioration models assume the opposite. This paper is based on an earlier work by Hwang for finding an optimal economic order quantity solution for items subject to amelioration in a Weibull distribution. Hwang solved the problem with a graphical procedure. In this paper, a rigorous analytical solution approach is derived for cases with different shape parameters. A detailed numerical example is also presented to illustrate the optimality of the derived solution.

1. Introduction

In view of the significant number of deterioration phenomena of inventory items in practice, much research has been performed to develop various deterioration models. Inventory models were proposed to deal with deteriorating items where the amount of stock decreases gradually over time [2–8]. A contrasting phenomenon, amelioration, has recently been proposed and studied [1,9]. Without considering the influence of demand, the definition for ameliorating items is that the amount of stock increases gradually during their holding period. Amelioration phenomena do exist in industries such as farming, fisheries, and poultry farming. Fast growing livestock such as ducks, pigs, broiler in poultry farms, hybrid fish in ponds and the culture of vegetables, fruits on farms are some examples. The phenomenon of amelioration is the opposite of deterioration and the modeling of it deserves an in-depth study.

Since a Weibull distribution can be used to effectively describe the life-span of a product by varying parameter values, it has been applied to the modeling of the product life cycle for inventory management in recent years. Jalan et al. [2], Chakrabarty et al. [3], and Wu et al. [4] investigated various deterioration inventory models. Chang and Dye [5], Wu [6,7], and Giri et al. [8] further extended the inventory model with deteriorating items in several directions. Hwang [1] was the first researcher to consider the inventory model with ameliorating items. Hwang [1,10] and Mondal et al. [9] developed the inventory models with ameliorating items under various formulations.

In Hwang's [1] EOQ model, instead of deriving an exact analytic solution, he used a graphical procedure of Gupta [11] to estimate the optimal cycle time. Contrary to his approach, an analytical framework is established in this paper to derive an optimal solution for the EOQ model. Finally, the same numerical example from Hwang [1] is adopted to illustrate the correctness of the proposed approach and to show the error in Hwang's solution.

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2. Notation and assumptions

This paper adopts the notation and expressions from Hwang [1] together with two auxiliary functions and several new expressions for discussion and analysis.

R	constant demand rate.
t	parameter to denote the time of amelioration.
C_o	ordering cost.
C_p	purchase cost.
C_a	amelioration cost, with $C_p > C_a$.
C_h	holding cost.
$A(t)$	amelioration rate with $A(t) = \alpha\,\beta t^{\beta-1}$ where α is the shape parameter and β is the scale parameter with $\beta > 0$.
$I(t)$	inventory level.
T	cycle time.
$TC(T)$	total cost per unit time that consists of ordering cost, purchasing cost, holding cost and the amelioration cost.
$f(T)$	auxiliary function with $\frac{\mathrm{d}TC(T)}{\mathrm{d}T} = \frac{R}{T^2} f(T)$.
$g(T)$	auxiliary function with $\frac{\mathrm{d}}{\mathrm{d}T} f(T) = \alpha\,\beta T^{\beta} \mathrm{e}^{-\alpha T^{\beta}} g(T)$.
$T^{\#}$	unique positive solution for $\frac{\mathrm{d}}{\mathrm{d}T} g(T) = 0$ with $T^{\#} = (2(1-\beta)/\alpha\beta)^{1/\beta}$.
T^{*}	optimal solution.

3. Review of past results

In Hwang [1], the initial inventory level, I_0, was first found. Then the inventory level, $I(t)$, for inventory model of ameliorating items with Weibull distribution is derived, as follows:

$$I_0 = R \int_0^T \mathrm{e}^{-\alpha\, t^{\beta}} \mathrm{d}t, \tag{1}$$

and

$$I(t) = \mathrm{e}^{\alpha\, t^{\beta}} \left(-R \int_0^t \mathrm{e}^{-\alpha\, x^{\beta}} \mathrm{d}x + I_0 \right). \tag{2}$$

The objective function, $TC(T)$, can be represented as follows:

$$TC(T) = I_0 \left(\frac{C_p - C_a}{T} + \frac{C_h}{2} \right) + RC_a + \frac{C_o}{T}. \tag{3}$$

Hwang [1] used a Taylor series expansion to approximate the exponential function in Eq. (1) resulting in:

$$I_0 = R \sum_{k=0}^{\infty} \frac{(-\alpha)^k\, T^{k\beta+1}}{k!\,(k\beta+1)}. \tag{4}$$

Eq. (4) was then substituted into Eq. (3) to compute the derivative of the object function, $\frac{\mathrm{d}TC(T)}{\mathrm{d}T}$, which yielded

$$\frac{\mathrm{d}TC(T)}{\mathrm{d}T} = (C_p - C_a)\; R \sum_{k=0}^{\infty} \frac{(-\alpha)^k\, k\beta}{k!\,(k\beta+1)} T^{k\beta+1} + \frac{C_h}{2} R \sum_{k=0}^{\infty} \frac{(-\alpha)^k\,(k\beta+1)}{k!} T^{k\beta} - \frac{C_o}{T^2}. \tag{5}$$

Eq. (5) was used to derive a functional relationship based on $\frac{\mathrm{d}TC(T)}{\mathrm{d}T} = 0$, resulting in

$$T = \frac{R}{C_o} \sum_{k=0}^{\infty} \frac{(-k)^k\, T^{k\beta-3}}{k!} \left[(C_p - C_a) \frac{k\beta\, T}{k\beta+1} + \frac{C_h}{2} \right]. \tag{6}$$

It was claimed that Eq. (6) can then be solved by a computer search method, a graphical procedure by Gupta [11], provided in the same paper [1].

From the differential equation for the inventory level, it is clear that

$$\frac{\mathrm{d}}{\mathrm{d}t} I(t) = \alpha\,\beta\, t^{\beta-1} I(t) - R$$

where the ending inventory level is zero as $I(T) = 0$. Solving the differential equation results in

$$I(t) = \mathrm{e}^{\alpha\, t^{\beta}} R \int_t^T \mathrm{e}^{-\alpha\, x^{\beta}} \mathrm{d}x. \tag{7}$$