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An interval programming approach for developing economic order quantity model with imprecise exponents and coefficients

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ABSTRACT

This paper develops an economic order quantity (EOQ) model with uncertain data. For modelling the uncertainty in real-world data, the exponents and coefficients in demand and cost functions are considered as interval data and then, the related model is designed. The proposed model maximises the profit and determines the price, marketing cost and lot sizing with the interval data. Since the model parameters are imprecise, the objective value is imprecise, too. So, the upper and lower bounds are specially formulated for the problem and then, the model is transferred to a geometric program. The resulted geometric program is solved by using the duality approach and the lower and upper bounds are found out for the objective function and variables. Two numerical examples and sensitivity analysis are further used to illustrate the performance of the proposed model.

1. Introduction

The economic order quantity (EOQ) is one of the most important problems in the lot sizing strategies. The producers always try to maximise their profits and find out the optimal values for various decision variables. In recent years, several researchers have introduced different models to optimise the EOQ models. In previous studies, to simplify the EOQ problem, it was assumed that the demand and production costs are fixed. As we know, in real-world applications, these assumptions are rarely satisfied. Therefore, development of the EOQ models in an uncertain environment is inevitable. Recent advances in the EOQ led to a good motivation for having more realistic optimal strategies.

Cheng [1] assumed that the production cost is affected by reliability and demand. Lee [2] considered the unit cost as a function of the order quantity. Lee and Kim [3], Cheng [4] and Jung and Klein [5] formulated a problem in which the unit price of the product is a function of product's demand. Others assumed that the unit production cost is affected by the production quantity [2,6–11]. Also, according to Jung and Klein [12], Liu [10], Lee et al. [13] and Lee [2,14] the demand is considered a function of the price. In some studies, the researchers have considered demand as a non-linear function of marketing and production cost. First, Lee and Kim [3] used this assumption and then, Esmaeili et al. [8] and Sadjadi et al. [9] considered that the demand is closely related to the selling price and expenditure per unit. Latter proposed models are non-linear and to solve and analyse them, geometric programming (GP) technique is applied. The GP technique has been improved by Avrielet et al. [15], Beightler and Philips [16], Duffin and Peterson [17], Duffin et al. [18] and Fang et al. [19].

Kochenberger [20] was the first person who used GP to solve the quantitative problems. Cheng [4] applied the GP technique to solve the EOQ model when production cost was affected by demand. Lee [14] assumed that the marketing

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expenditure, price and production volume were deterministic variables. Lee and Kim [3] examined the effects of integrated production and marketing decisions on maximising the firm's profit. They concluded that the production unit cost is affected by the demand through an economy of scale effect. Also, in their study, the demand is assumed as a function of price and marketing expenditure.

In the EOQ model, the value of exponents and scaling constants depends on decision-makers and scale of economics. It is clear that the final solution changes when the values of exponents and coefficients change. Considering the uncertainty in data, three approaches namely sensitivity analysis, fuzzy programming and interval programming are applied to find out the optimal solution. In sensitivity analysis, one parameter is changed at a time and the optimal value is determined. The sensitivity analysis can be found in Refs. [14,21–23]. In many problems, using the sensitive analysis will be time consuming. Therefore, other approaches are usually used for modelling uncertainty in data. Guiffrida [24] reviewed the application of fuzzy set theory in inventory management. There are several studies which take the fuzzy approach to model the uncertainty in the EOQ model. Roy and Maiti [25] developed a fuzzy EOQ model with limited capacity and solved it by both fuzzy non-linear programming and GP techniques. In their study, the demand is related to the unit price and the set-up cost varies with the quantity produced/purchased. Yao and Lin [26] calculated the optimal fuzzy profit using the fuzzy price function and linear cost function. Panda et al. [27] considered the cost function as a fuzzy number. Liu [28] extended a solution method to derive the fuzzy profit with fuzzy demand and fuzzy unit cost. Sadjadi et al. [29] proposed a new possibilistic model which makes it easy to build the overall model based on experts' opinions. The proposed model is formulated in fuzzy GP and using the optimisation techniques, the lower and upper bounds are calculated for objective function. The fuzzy inventory problem is transformed into a pair of two-level mathematical programs and the GP technique is applied to solve it. The disadvantage of the fuzzy approach lies in its inability to represent the exact nature of the uncertainty and the results could depend on the fuzzification approach [30]. Under an uncertain environment, the imprecise parameters are frequently represented by interval data [31,32]. Interval programming is a powerful approach for solving problems with inexact and imprecise parameters. Liu [10] developed a solution method for the EOQ model with interval coefficients. He formulated a pair of mathematical programming model to derive the upper and lower bounds of the profit. Also, he developed a solution method to calculate the bounds of the objective function in the interval GP [31]. In Liu's studies [31,32], the objective function of the model was posynomial.

In this paper, it is assumed that the demand is a non-linear function of the price and marketing expenditure and also, the cost is affected by lot sizing. The exponents and coefficients of the demand and quantity discounts are considered interval data. Then, an interval programming model is used to find out the optimal solution of the model. In previous studies, only the coefficients (the scaling constant) had been taken as interval values and the demand was dependent only on price. In addition, in this paper, Liu's [10] approach is developed and a new approach is proposed to find out the upper and lower bounds of the EOQ model.

The rest of this paper is organised as follows. In the next section, the problem statement and the proposed model are presented. In Section 3, the upper and lower bounds of the EOQ model are found, and then the model is converted to a dual GP model. In Section 4, two numerical examples are solved and the sensitivity analysis is presented. Finally, the conclusion of this paper is presented in Section 5.

2. Problem statement

This section introduces the notation and formulation of the proposed model. All decision variables, input parameters and assumptions will be stated.

2.1. Decision variables

P selling price per unit
M marketing expenditure per unit
Q production lot size (units)

2.2. Input parameters

k scaling constant for demand function ($k > 0$)
c production cost per unit
i inventory cost per unit
r scaling constant for unit production cost
a set-up cost
n production rate
α price elasticity of demand function
β lot size elasticity of production unit cost ($0 < \beta < 1$)
γ marketing expenditure elasticity ($0 < \gamma < 1$)