Applied Mathematical Modelling 37 (2013) 9698–9706

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An Economic order quantity model for Items with Three-parameter Weibull distribution Deterioration, Ramp-type Demand and Shortages

S.S. Sanni *, W.I.E. Chukwu

Department of Statistics, Faculty of Physical Sciences, University of Nigeria, Nsukka, Enugu, Nigeria

A R T I C L E   I N F O

Article history:
Received 30 May 2012
Received in revised form 16 April 2013
Accepted 21 May 2013
Available online 5 June 2013

Keywords:
Inventory model
Ramp type demand
Differential equation
Optimal policy
Instantaneous rate function

A B S T R A C T

In this paper, we develop an economic order quantity inventory model for items with three-parameter Weibull distribution deterioration and ramp-type demand. Shortages are allowed in the inventory system and are completely backlogged. The demand rate is deterministic and varies with time up to a certain point and eventually stabilized and becomes constant. The instantaneous rate of deterioration is an increasing function of time. We provide simple analytical tractable procedures for deriving the model and give numerical examples to illustrate the solution procedure. Our adoption of ramp-type demand reflects a real market demand for newly launched product.

1. Introduction

Inventory control policy is concerned basically with two decisions ; "How much to order (produce or purchase) to replenish the inventory of an item" and "When to order so as to minimize the total cost" [1]. Several inventory models have been developed to answer the above questions. The economic-order-quantity model, hereafter EOQ, was originally developed by Ford W. Harris in 1913, but R. H. Wilson a consultant who applied it extensively, is given credit for his in-depth analysis [2]. The basic EOQ model assumes a constant demand rate and an infinite planning horizon, there is no deterioration of inventory and replenishment is instantaneous i.e. the lead time is zero. These assumptions restrict the applicability of the classical EOQ model. In order to make the basic EOQ model more realistic, many researchers have extended Wilson's EOQ model by considering time/price varying demand pattern and deterioration rate. It ispertinent to note that the depletion of any inventory is due mainly to demand and partly to deterioration of the item.

In what follows we give definition of terms central to the paper. In particular, we define deterioration, ramp-type function and Weibull function.

Definition 1.1. Deterioration is defined as decay, damage, change or spoilage that prevents items from being used for its original purpose. Some examples of items that deteriorate are fashion goods, foods, mobile phones, chemicals, automobiles, drugs, etc.

Definition 1.2. A random variable Y (e.g. the time to deterioration of an item) is said to have a Weibull distribution if its density is given, for some parameters $\alpha > 0, \beta > 0,$ and γ, by $f(y) = \alpha\beta(y-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}}, y > 0$. The parameters α, β, and γ

* Corresponding author. Tel.: +234 8035813754.
E-mail addresses: sanni_smaila@yahoo.com (S.S. Sanni), chukwuagusi@yahoo.com (W.I.E. Chukwu).

0307-904X/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2013.05.017

are the the scale, shape and location parameters of the distribution, respectively. For time to deterioration data, the scale parameter, α represents the characteristic rate of deterioration of items, the shape parameter, β is a measure of the spread of time-to-deterioration and γ is the minimum time such that $y \geqslant \gamma$. A negative γ may indicate that deterioration has occurred prior to the beginning of the inventory, namely during production, in transit, or prior to actual storage while $\gamma = 0$ reduces the density to the case of two-parameter Weibull distribution. The scale and the location parameters have the same unit as T (i.e. cycle length in time) while the shape parameter is dimensionless. Weibull model is widely used today for failure and survival analysis.

Definition 1.3. Let $\Re$ be the set of real numbers. A function $g : \Re \to \Re$ defined by $g(x) = \begin{cases} h(x), & x < \mu \\ h(\mu), & x \geqslant \mu \end{cases}$ is called a ramp function. In particular, $g(x) = a[x - (x-\mu)H(x-\mu)]$ where $H(x-\mu) = \begin{cases} 0, & x < \mu \\ 1, & x \geqslant \mu \end{cases}$ and $a \neq 0, \mu > 0$ is a ramp-type function. A variety of ramp-type functions have evolved from the above definition.

Inventory models for deteriorating items have been widely studied by researchers. Inventory problem for deteriorating items was first studied by Whitin [3], he considered fashion items decaying at the end of the planning horizon. Wagner and Whitin[4] developed a dynamic version of the classical EOQ model. Thereafter, Ghare and Schrader[5] developed a model for exponentially deteriorating inventory. They proposed, explicitly for the first time, the differential equation governing the variation in the inventory system ; $dI(t)/dt + \theta I(t) = -D(t)$. Donaldson[6] provided a somehow complicated analytical solution procedure for the basic inventory policy for the case of positive linear trend in demand. Silver[7] formulated a heuristic for deteriorating inventory model with time-dependent linear demand. Deb and Chuadhuri[8] extended the deteriorating inventory model with linear demand by incorporating shortages in the inventory. Covert and Philip [9] presented an inventory model where the time to deterioration is described with two-parameter Weibull distribution. Philip[10] generalized the model in [9] by considering a three-parameter Weibull distribution deterioration, no shortages and a constant demand. Chakrabarty *et al.* [11] proposed an EOQ model with three-parameter Weibull distribution deterioration, shortages and linear demand rate and obtained infinite series representation for the initial inventory level and the average total cost equation. Ghosh and Chaudhuri [12] developed an inventory model for two-parameter Weibull deteriorating items, with shortages and quadratic demand rate and gave infinite series representation for the initial inventory level and the total average variable cost equation. Sanni [13] developed an inventory model with three-parameter Weibull deterioration, shortages and quadratic demand rate. He derived explicit equations for the initial inventory level and the average total cost by approximating exponential functions by the first two terms of the Tailor series. An order-level inventory model for deteriorating items with ramp type demand rate was discussed by Mandal and Pal [14]. They considered an EOQ model for items with constant rate of deterioration, ramp-type demand rate and no shortage allowed in the system and obtained an approximate solution for the EOQ. This work was extended by Wu and Ouyang [15] by considering two types of shortages in the model: model that starts with stock and model that starts with shortages and obtained optimal replenishment policy for the different cases. This model was further generalized by Samanta and Bhowmick [16] by taking two parameter Weibull distribution to represent the time to deterioration and allowed shortages in the inventory.They studied two cases; where the inventory starts with shortages and the case where the system starts without shortages and derived the EOQ for the respective systems. For some literature on deteriorating inventory models, see [12,17,18].

In this paper we consider the problem of finding optimal replenishment policy for an inventory system which holds items with three parameter Weibull Distribution deterioration, ramp type demand rate and shortages are allowed and completely backlogged. The research focus of this paper is to develop a mathematical model for the system, provide an optimal replenishment policy for the model and establish the necessary and sufficient conditions for the optimal policy. The Weibull distribution is suitable for items whose rate of deterioration increases with time and the location parameter γ, in the three-parameter Weibull distribution, is used here to depict the item shelf-life; an important feature of most deteriorating items. The ramp type demand rate describes the demand of products, such as fashion goods, electronics, automobiles, etc, for which the demand increases as they are launched into the market and after some time the demand stabilizes and becomes constant.

2. Assumptions and notation

The mathematical model in this work is developed on the basis of the following assumptions and notation.

Notation

- C_1 : inventory holding cost per unit per unit time.
- C_2 : shortage cost per unit per unit time.
- C_3 : ordering cost per order.
- C_4 : unit cost.
- $D(t)$: demand rate at any time $t \geqslant 0$.
- T : cycle length.
- I_o : size of the initial inventory.