Omega 40 (2012) 328–335

Contents lists available at SciVerse ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Economic order quantity model with trade credit financing for non-decreasing demand

Jinn-Tsair Teng [a], Jie Min [b,*], Qinhua Pan [c]

[a] Department of Marketing and Management Sciences, The William Paterson University of New Jersey, Wayne, NJ 07470-2103, USA
[b] Department of Mathematics and Physics, Anhui University of Architecture, Hefei, PR China
[c] Department of Economics and Finance, School of Economics and Management, Tongji University, Shanghai, PR China

ARTICLE INFO

Article history:
Received 9 November 2010
Accepted 2 August 2011
This manuscript was processed by Associate Editor Dolgui
Available online 10 August 2011

Keywords:
Inventory
Trade credit
Increasing demand
Finance

ABSTRACT

Researchers in the past have established their inventory lot-size models under trade credit financing by assuming that the demand rate is constant. However, from a product life cycle perspective, it is only in the maturity stage that demand is near constant. During the growth stage of a product life cycle (especially for high-tech products), the demand function increases with time. To obtain robust and generalized results, we extend the constant demand to a linear non-decreasing demand function of time. As a result, the fundamental theoretical results obtained here are suitable for both the growth and maturity stages of a product life cycle. In addition, we characterize the optimal solutions and obtain conclusions on important and relevant managerial phenomena. Lastly, we provide several numerical examples to illustrate the proposed model and its optimal solution.

1. Introduction

Under the classical inventory economic order quantity (EOQ) model, it is assumed that the retailer must pay for items upon receiving them. In practice, suppliers frequently offer retailers a time period for payment of the amount owed. Usually, there is no interest charge if the outstanding amount is paid within this permissible delay period. However, if the payment is unpaid in full by the end of the permissible delay period, interest is charged on the outstanding amount. The permissible delay period for payment produces two benefits to the supplier: (1) it attracts new buyers who consider it a type of price reduction, and (2) it may be applied as an alternative to a discounted price because it neither illicits price cutting from competitors, nor does it introduce permanent price reductions. On the other hand, the policy of granting credit terms adds an additional cost to the supplier as well as an additional dimension of default risk.

Goyal [1] developed an EOQ model under the conditions of a permissible delay in payments, and ignored the difference between the selling price and the purchase cost. Although Dave [2] corrected Goyal's model by addressing the fact that the selling price is necessarily higher than its purchase cost, Dave's viewpoint failed to draw sufficient attention from recent researchers. Shah [3] considered a stochastic inventory model when delays in payments are permissible. Aggarwal and Jaggi [4] extended Goyal's model to consider the deteriorating items. Jamal et al. [5] further generalized Aggarwal and Jaggi's model to allow for shortages. Hwang and Shinn [6] added the pricing strategy to the model, and developed the optimal price and lot-sizing for a retailer under the condition of a permissible delay in payments. Chung [7] developed an alternative approach to determine the economic order quantity under the condition of trade credit being granted. Teng [8] amended Goyal's model by considering the difference between unit price and unit cost, and found it to be of economic sense for a well-established buyer to order smaller quantities and avail itself of the benefits of the permissible delay more frequently. Chang et al. [9] developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Chung and Huang [10] developed an economic production quantity model (EPQ) for a retailer where the supplier offers a permissible delay in payments. Huang [11] extended Goyal's model to develop an EOQ model in which the supplier offers the retailer the permissible delay period M (i.e., the upstream trade credit), and the retailer in turn provides the trade credit period N (with $N \le M$) to its customers (i.e., the downstream trade credit). Recently, Teng and Goyal [12] addressed the shortcoming of Huang's model and proposed a generalized formulation. Many related articles can be found in Chang and Teng [13], Chung [14], Chung and Liao [15], Goyal et al. [16], Huang [17,18], Huang and Hsu [19], Liao et al. [20], Ouyang et al. [21,22], Shinn and Hwang [23], Teng and Chang [24], Teng et al. [25–27], and their references.

All the above researchers established their EOQ or EPQ inventory models under trade credit financing by assuming that

* Corresponding author. Tel.: +86 551 3828230.
E-mail address: minjie@aiai.edu.cn (J. Min).

0305-0483/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.omega.2011.08.001

the demand rate is constant. However, from a product life cycle perspective, it is only in the maturity stage that demand is near constant. During the growth stage of a product life cycle, the demand function increases with time. To obtain robust and generalized results, we propose in this paper to extend the constant demand to a linear non-decreasing demand function of time, which is suitable not only for the growth stage but also for the maturity stage of a product life cycle. In the process, we establish some fundamental theoretical results, and obtain important and relevant conclusions on managerial phenomena. Lastly, we provide several numerical examples to illustrate the proposed model and its optimal solution. It is notable that Hsieh et al. [28] recently generalized the demand rate for an EOQ model with upstream and downstream trade credits to an increasing function of time, and proved that a unique global minimum cost per unit of time exists. However, they assumed that both the first derivative and the second derivative of the demand rate must be great than zero, which excluded not only a constant demand but also a linearly increasing demand. As a result, the results reached in our paper are not covered by Hsieh et al.'s conclusions.

2. Notation and assumptions

The following notation and assumptions are used in this paper.

2.1. Notation

K	ordering cost per order
c	unit purchasing cost
s	unit selling price (with $s>c$)
h	unit stock holding cost per unit of time (excluding interest charges)
I_e	interest, which can be earned per \$ per unit of time by the retailer
I_c	interest charges per \$ in stocks per unit of time by the supplier
M	the retailer's trade credit period offered by supplier in years
T	inventory cycle length (decision variable)
Q	the retailer's order quantity at time 0
$\Pi(T)$	the retailer's profit function per unit of time
T_1	the optimal cycle time with trade credit financing, which is less than or equal to M
T_2	the optimal cycle time with trade credit financing, which is greater than or equal to M
T_e	the optimal cycle time without trade credit financing in the classical EOQ model
Q_e	the optimal order quantity without trade credit financing in the classical EOQ model

2.2. Assumptions

1. During the growth stage of a product life cycle especially for a high-tech product, the demand rate $D(t)$ can be given by
$$D(t)=a+bt, \tag{1}$$
where a and b are non-negative constants, and t is within a positive time frame.
2. The objective here is to maximize the profit per unit of time for the first replenishment cycle. In reality, we make an initial solution under initial information, and then change the solution whenever information is changed. Hence, after obtaining the initial optimal cycle time I_1 based on $D(0)=a$, we must reevaluate the situations to see whether the model still holds or not. If so, we can reuse the same method to obtain the next optimal cycle time I_2 based on the new $D(0)=a+bI_1$. Otherwise, we cannot apply it. Since the problem with constant or ramp-type demand has been solved in literature such as Teng [8] and Skouri et al. [29], we focus on the problem with increasing demand only.
3. The lead time is negligible.
4. Shortages are not allowed to occur.
5. The retailer would settle the account at $t=M$ and pay for the interest charges on items in stock with rate I_c over the interval $[M, T]$ as $T\geq M$. Alternatively, the retailer settles the account at $t=M$ and is not required to pay any interest charge for items in stock during the whole cycle as $T\leq M$.
6. The retailer can accumulate revenue and earn interest from the beginning of the inventory cycle until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period from $t=0$ to $t=M$ with rate I_e under the trade credit conditions.

Given the above, it is possible to formulate a mathematical inventory EOQ model with trade credit financing.

3. Mathematical model

Based on the above assumptions, the inventory system can be considered as follows. At the beginning (e.g., at time $t=0$), the retailer orders and receives Q units of a single product from the supplier. The inventory level is depleted gradually in the interval $[0, T]$ due to demand from customers. At time $t=T$, the inventory level reaches zero. Hence, the variation of inventory level, $I(t)$, with respect to time can be described by the following differential equation:

$$\frac{dI(t)}{dt}=-D(t)=-(a+bt), \quad 0\leq t\leq T, \tag{2}$$

with the boundary condition $I(T)=0$.

The solution to above differential equation is

$$I(t)=a(T-t)+\frac{1}{2}b(T^2-t^2), \quad 0\leq t\leq T. \tag{3}$$

As such, the retailer's order size per cycle is

$$Q=I(0)=aT+\frac{1}{2}bT^2. \tag{4}$$

The elements comprising the retailer's profit function per cycle are listed below:

(a) The ordering cost $=K$.
(b) The holding cost (excluding interest charges) $=h\int_0^T I(t)dt = h((1/2)aT^2+(1/3)bT^3)$.
(c) The purchasing cost $=cQ=c(aT+(1/2)bT^2)$.
(d) The sales revenue $=sQ=s(aT+(1/2)bT^2)$.
(e) From Assumption 5, two alternative cases arise in relation to interest payable.
Case 1: $T\leq M$.
Since the cycle time T is shorter than the credit period M, there is no interest paid for financing the inventory in stock. Therefore, the interest payable in this case is zero.
Case 2: $T\geq M$.
When the credit period M is shorter than or equal to the replenishment cycle time T, the retailer begins to pay interest for the items in stock after time M with rate I_c. Hence, the interest payable is

$$cI_c\int_M^T I(t)dt = cI_c\left[\frac{1}{2}a(T-M)^2+\frac{1}{2}bT^2(T-M)-\frac{1}{6}b(T^3-M^3)\right].$$