Economic order quantity model for deteriorating items with planned backorder level

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ABSTRACT

In this study, a deteriorating inventory problem with and without backorders is developed. From the literature search, this study is one of the first attempts by researchers to solve a deteriorating inventory problem with a simplified approach. The optimal solutions are compared with the classical methods for solving deteriorating inventory model. The total cost of the simplified model is almost identical to the original model.

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1. Introduction

The classical Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) have been extended by many researchers for decades. Approaches without using derivative have attracted much attention in recent years. Grubbström and Erdem [1] and Cárdenas-Barrón [2] were among the first researchers to derive EOQ and EPQ with backorders without derivatives, respectively. Later, Yang and Wee [3] and Cárdenas-Barrón et al. [4] developed a method without derivatives to solve an integrated vendor–buyer inventory system. EOQ problem with temporary sale price without derivative was developed by Wee et al. [5]. Chang et al. [6] used algebraic approaches to solve EOQ and Economic Production Quantity (EPQ) model with shortage. Sphicas [7] solved EOQ and EPQ with linear and fixed backorder cost. Cárdenas-Barrón [8] solved the EPQ with rework process using the algebraic method. Minner [9] introduced the cost-difference comparison method for solving the EOQ problem. Wee and Chung [10] extended the research by simplifying the solution procedure. Cárdenas-Barrón [11] applied the nonderivatives method to solve inventory problem in multi-stages multi-customers supply chain. Teng [12] developed arithmetic–geometric-mean-inequality theorem to solve EOQ problem without derivatives. He applied the theorem to EOQ with/without backorder, and EPQ with backorder. The arithmetic–geometric-mean-inequality approach was used by Ouyang et al. [13] and Teng et al. [14]. Ouyang et al. [13] solved an economic order quantity model with partially permissible delay in payments and defective items, and Teng et al. [14] solved an integrated vendor–buyer inventory model considering backorders. Cárdenas-Barrón [15] developed the EOQ/EPQ models using the arithmetic–geometric mean and Cauchy–Bunyakovsky–Schwarz inequality. A complete review of several optimization approaches used in inventory can be found in [16].

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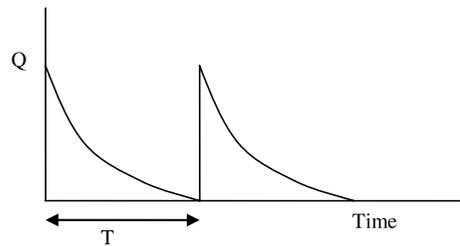*E-mail address:* weehm@cycu.edu.tw (H.M. Wee).

Fig. 1. Original deteriorating inventory level.

Some goods (i.e. milk, fruits, vegetables, among others) deteriorate with time. Deterioration means decay, evaporation, obsolescence, loss of quality or marginal value of a commodity. Obviously, deterioration decreases the usefulness of the good from its original condition. The longer the goods are kept in inventory, the higher the deteriorating cost. The development of deteriorating inventory model has been done by many researchers. See, for example, [17–25]. Most of research above cannot derive a close form solution since it is not possible to derive a close form solution for most deteriorating items inventory model. Goyal and Giri [26] presented a complete review of the deteriorating inventory literature since the early 1990s. Zipkin [27] solved deteriorating inventory problem with financing cost numerically using computer. None of the above researchers use algebraic approach to consider deteriorating item inventory. Our method can be used to develop in a simple way more complex inventory models considering imperfect production process (i.e. [28–30]) or imperfect quality products (i.e. [31,32]).

In this paper, we proposed an EOQ model for deteriorating item problem. The solutions are derived using modified cost-different comparison used in [33] for nondeteriorating items. Section 2 shows notations and the problem definition. Models development of simple EOQ solution for deteriorating item and the deteriorating inventory problem with backorder are shown in Section 3 and some conclusions are derived in Section 4.

2. Notations and problem definition

Notations:
A setup cost ($ per order)
h inventory holding cost ($ per unit and unit time)
$\hat{\pi}$ backorder cost ($ per unit and unit time)
D demand rate (units per time)
θ deterioration rate
r optimal fill rate
n batch number
T replenishment time (units of time)
T_1 nonbackorder time (units of time), where $T_1 = r * T$
Q^* optimal order quantity (units)
B^* backorder level quantity (units)
$TC(T)$ total cost in T replenishment time
$TC_u(i, T)$ total unit cost in i batch and T replenishment time
$TC(r, n, T)$ total unit cost in r optimal fill rate, n batch and T replenishment time

The problem of deteriorating inventory model is similar as the basic inventory problem, except that inventory is assumed to deteriorate at a constant rate. Similar to the basic model, the demand rate in our model is assumed to be constant. When items are needed, they can be delivered instantaneously. The cost of deteriorated items is constant and equal to unit cost. It is also assumed that the deteriorated items are not repaired or replaced by a good unit. The number of units will be treated as a continuous variable. Similar to classic EOQ models, the batch size for each replenishment is the same.

3. Models development

In this paper, two EOQ models for deteriorating items without backorder and with backorder are developed and details of calculation methods are presented in the following sections.

3.1. EOQ without backorder

The deteriorating inventory level is illustrated in Fig. 1.
The inventory level can be denoted by the following equation:

$$\frac{dI}{dt} + \theta I_t = -D, \quad 0 \leq t \leq T. \tag{1}$$