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Applied Mathematical Modelling 31 (2007) 10–17
APPLIED MATHEMATICAL MODELLING
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An EPQ model with setup cost and process quality as functions of capital expenditure

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Received 1 May 2004; received in revised form 1 December 2005; accepted 9 March 2006
Available online 10 July 2006

Abstract

This paper considers an economic production quantity (EPQ) model with imperfect production processes, in which the setup cost and process quality are functions of capital expenditure. The mathematical model is derived to investigate the effects of an imperfect production process on the optimal production cycle time when capital investment strategies in setup reduction and process quality improvement are adopted. An efficient procedure is developed to find the optimal production run length, setup cost and process quality. Finally, a numerical example is provided to illustrate the theoretical results. Some managerial implications are also included.

Keywords: Inventory; Production cycle; Imperfect production process; Investment strategy

1. Introduction

The economic production quantity (EPQ) model has been widely used in practice because of its simplicity. In the past several decades, numerous research efforts have been undertaken to extend the basic EPQ model by relaxing various assumptions so that the model conforms more closely to real-world situations. The classical EPQ model assumes that the production facility is failure free and all the items produced are of perfect quality, and that quality level is fixed at an optimal level. However, in real production environment, it can often be observed that the product quality is not always perfect and usually depends on the state of the production process. In addition, there are defective items being produced due to imperfect production processes. The defective items must be rejected, repaired and reworked, and thus substantial costs incur. Hence, the inventory policy determined by the conventional model might be inappropriate. To study the effects of imperfect quality to lot sizes, several authors have developed various models involving the quality-related issues. Rosenblatt and Lee [1] and Porteus [2] initially studied the effect of process deterioration on the optimal production cycle time or EPQ. Specifically, Rosenblatt and Lee [1] assumed that the elapsed time until the production process shift is

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doi:10.1016/j.apm.2006.03.034

exponentially distributed and derived an approximated optimal production run length. Porteus [2] made a similar study of a production process, but assumed that the process goes out-of-control with a given probability each time an item is produced. Rosenblatt and Lee [1] and Porteus [2] both found that the optimal EPQ is smaller than the EPQ in the classical model because smaller lot produces fewer defective items. Kim and Hong [3] extended the work of Rosenblatt and Lee [1] by assuming that an elapsed time until shift is arbitrarily distributed. Lee et al. [4] studied investment in quality improvement in order to reduce the proportion of defective items and affect the inventory cost, profit loss, and internal and external failure costs. Recently, Hou and Lin [5] studied the effects of an imperfect production process on the optimal production run length when capital investment in process quality improvement is adopted. The optimal lot sizing and capital investment are appropriately determined, and relevant critical performance measures are also discussed. Lee [6] presented a cost/benefit model for investments in inventory and preventive maintenance in an imperfect production system in order to increase product and service quality. Other related studies on lot sizing with deteriorating production process include Cheng [7], Khouja and Mehrez [8], Makis [9] and references therein.

On the other hand, the setup cost is usually treaded as a constant in the classical EPQ model. However, in practice, setup cost can be controlled and reduced through various efforts such as worker training, procedural changes and specialized equipment acquisition. The benefits of reduced setups are well documented (e.g., see Hong and Hayya [10]). For example, faster changeovers have been associated with lower inventory, faster throughput, shorter lead time, improved quality, and lower unit cost. Quick setups are also considered an important element for successfully implementing just-in-time (JIT) production or time-based competition. Therefore, for attaining production system efficiency, reduced lot sizes alone are not sufficient, unless accompanied by corresponding setup cost reduction and quality improvement. Therefore, considerable attention has been paid to the optimal lot sizing and investments in setup cost reduction and quality improvement. Porteus [2] introduced different options for investing in setup cost reduction and quality improvement. Following Poretus, other authors, such as Keller and Noori [11], Hong et al. [12], and Hong and Hayya [13], studied the economic benefits of reducing setup cost and improving process quality by simultaneously investing in new technology. Affisco et al. [14] investigated the investments in setup cost reduction and quality improvement for a joint supplier–customer system with defects produced at a known constant rate. Lin and Hou [15] considered an inventory system with random yield in which both the setup cost and yield variability can be reduced through capital investment. They presented an iterative solution procedure to find the optimal order quantity and reorder point and then the optimal setup cost and yield standard deviation. Recently, Hong [16] extend the work of Goyal [17] and Rosenblatt and Lee [1] to consider the determination of production cycles, procurement schedules, and joint investments in setup reduction and process quality improvement for a production system with imperfect production processes. However, for mathematical simplicity and practical convenience, the majority of above-mentioned literature (see [16,12,10,8,2,1]) used a truncated Taylor serried expansion for the exponential term to present the annual cost function. Consequently, they only derived an approximated solution which was not an optimal one and resulted in overvalue of total annual cost. In addition, the EPQ proposed in the literatures are not always a good substitute for the cost minimization. Hence, their solution procedures may be inappropriately to find the optimal lot sizes and the investments optimally for setup reduction and quality improvement when based on the actual cost function of inventory systems.

Based on the above arguments, the purpose of this paper here is to develop a cost minimization model with capital investments to explicitly obtain the optimal lot sizing and investments in setup cost reduction and process quality improvement. To our knowledge, the logarithmic investment function which has been used in previous researches by Porteus [2], Billington [18], Paknejad et al. [19], Sarker and Coates [20], Hofmann [21], Chung et al. [22], and Hou and Lin [5], etc., is consistent with the Japanese experience as reported in Hall [23]. Therefore, this article assumes that the relationship between setup cost reduction (or process quality improvement) and capital investments can be described by the logarithmic investment function. The proposed model can be considered as an extension of the previous inventory models by investigating the advantages of capital investments in quality improvement and reducing setup costs. A simple and accurate algorithm is presented to locate the optimal production run length, and then to find the optimal setup cost and optimal process quality simultaneously. The results are compared with no-investment option solution as reported in Rosenblatt and Lee [1] and Kim and Hong [3]. Finally, a numerical example is given to illustrate the results obtained and assess the cost savings by adopting capital investments.