ELSEVIER

Int. J. Production Economics 83 (2003) 223–231

international journal of production economics

www.elsevier.com/locate/dsw

Probabilistic multi-item inventory model with varying order cost under two restrictions: A geometric programming approach

M.O. Abuo-El-Ata, Hala A. Fergany*, Mona F. El-Wakeel

Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt

Received 19 October 2001; accepted 26 June 2002

Abstract

A probabilistic multi-item inventory model with varying order cost and zero lead time under two restrictions is treated in this paper under the following assumptions: (1) the maximum inventory level of each item is a constant multiple of the average quantity ordered; (2) the order cost is a continuous increasing function of the replenishment quantity, which itself is proportional to some number of periods covered by the replenishment quantity. The constant of proportionality is the average demand per period. The expected total cost of inventory management is composed of three components: the average purchase cost, which is a constant that does not enter into the optimization consideration; the expected ordering cost, and the expected holding cost. No shortages are to be allowed. An analytical solution of the optimal number of periods N_r^* (rounded integer) and the optimal maximum inventory level is derived using a geometric programming approach. There are four special cases corresponding to the three possible relaxations of the constraints plus the case of the classical probabilistic model of constant procurement cost combined with the absence of the constraints. Also, an illustrative numerical example is added with some graphs.

Keywords: Probabilistic model; Multi-item; Varying order cost; Geometric programming and optimal number of periods

1. Introduction

Most of the literature dealing with probabilistic inventory models assumes that the demand rate is probabilistic since the probability distribution of the future demand rate rather than the exact value of demand rate itself, is known. Most of the probabilistic inventory models assume that the units of cost are constant and independent of the number of periods. Unconstrained probabilistic inventory models with constant unit of costs have been treated by Gupta and Hira (1994), Hadley and Whitin (1963), and Taha (1997).

Fabrycky and Banks (1967) studied the probabilistic single-item, single-source (SISS) inventory system with zero lead time, using the classical optimization. Recently, Abou-El-Ata and Mousa (1998) studied the

*Corresponding author.

0925-5273/03/$ - see front matter
PII: S0925-5273(02)00327-4

deterministic multi-item inventory model with varying order cost under two restrictions. Also, Fergany (1999) discussed the multi-item inventory system with both demand-dependent unit costs and varying leading time using the Lagrangian multiplier.

In this research, we investigate a probabilistic multi-item, single-source (MISS) inventory model with varying order cost under two restrictions, one of them is on the expected order cost and the other on the expected holding cost. The optimal number of periods N_r^* and the optimal expected total cost min $E(\mathrm{TC})$ are obtained. Also, some special cases are deduced and an illustrative numerical example is added with some graphs.

2. Model development

The following notations are adopted for developing our model:

$C_{\mathrm{p}r}$	the purchase cost of the rth item
$C_{\mathrm{o}r}(N_r)$	the varying order cost of the rth item per cycle
$C_{\mathrm{h}r}$	the holding cost of the rth item per period
D_r	a random variable demand rate of the rth item per period
$f(D_r)$	the probability density function of the demand rate
$E(D_r)$	the expected value of the demand rate
$E(Q_r)$	the expected order quantity of the rth item
$Q_{\mathrm{m}r}$	the maximum inventory level of the rth item
N_r	the number of periods of the rth item (a decision variable), and a review of the stock level of the rth item is made every N_r periods, "$E(Q_r) = N_r\,E(D_r)$"
K_1	the limitation on the expected order cost
K_2	the limitation on the expected holding cost
$E(\mathrm{TC})$	the expected total cost function

The following assumptions are made for developing our mathematical model:

(1) Assume that the maximum order quantity which equals the maximum inventory level is related to the expected order quantity during the cycle by a relational function just mentioned as $g(N)$. Then for $g(N) = a$ where a is a constant, we get

$$Q_{\mathrm{m}r} = a\,E(Q_r),$$

where $a > \frac{1}{2}$. The process is exhibited graphically in Fig. 1.

The expected level of inventory $\bar{I}$, is given by

$$\bar{I} = N_r\left(Q_{\mathrm{m}r} - \frac{E(Q_r)}{2}\right)$$

which yields

$$E(\mathrm{HC}) = \frac{C_{\mathrm{h}r}\bar{I}}{N_r} = \frac{C_{\mathrm{h}r}E(D_r)N_r b}{2}, \quad b = 2a - 1.$$

(2) The order cost per unit is a continuous increasing function of the expected number of periods, N_r, which takes the following form:

$$C_{\mathrm{o}r}(N_r) = C_{\mathrm{o}r}N_r^{\beta}, \tag{1}$$