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ORIGINAL ARTICLE

Constrained backorders inventory system with varying order cost: Lead time demand uniformly distributed

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Abstract This paper discusses the probabilistic backorders inventory system when the order cost unit is a function of the order quantity. Our objective is to minimize the expected annual total cost under a restriction on the expected annual holding cost when the lead time demand follows the uniform distribution. Then some special cases are deduced and an illustrative numerical example with its graphs is added.

1. Introduction

Many authors including Feldman (1978), Richards (1975) and Sahin (1979) have studied continuous review inventory models with constant units of cost and stationary distributions of inventory level. The inventory models under continuous review with stationary distribution of inventory level, or inventory position in the case of positive lead-time, have been derived using renewal theory as in Arrow et al. (1958). In addition, Ben-Daya and Abdul (1994) examined unconstrained inventory model with constant units of cost, demand follows a normal distribution and the lead-time is one of the decision variables.

Taha (1997) treated unconstrained probabilistic inventory problems with constant units of cost. Hadley and Whitin (1963) discussed probabilistic continuous review inventory models with constant units of cost and the lead-time demand is a random variable. Their work gives heuristic approximate treatment for each of the backorders and the lost sales cases. Fabrycky and Banks (1967) studied the probabilistic single-item, single source (SISS) inventory system with zero lead-time, using the classical optimization. Abou-El-Ata et al. (2003) introduced a probabilistic multi-item inventory model with varying order cost; zero lead-time demand under two restrictions and no shortage are to be allowed. Fergany and El-Wakeel (2006a,b), applied several continuous distributions for constrained probabilistic lost sales inventory models with varying order cost using Lagrangian method. Recently, Kotb and Fergany (2011) deduced multi-item EOQ model with varying holding cost using geometric programming approach.

This paper considering the backorders inventory model with varying order cost, a restriction on the expected annual holding cost and the lead-time demand follows Uniform distribution. The policy variables of this model are the order quantity and the reorder point, which minimize the annual total cost. Finally, two special cases are deduced, which have been previously published and a numerical illustrative example is added with its graphs.

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2. Assumptions and notations

The following assumptions are usually made in the simple treatments for developing the mathematical model:

1. The reorder point r is positive.
2. The demand is a random variable with known probability.
3. An order quantity of size Q per cycle is placed every time the stock level reaches a certain reorder point r.
4. Assume that the system repeats itself in the sense that the inventory position varies between r and $r + Q$ during each cycle.

The following notations are adopted for developing our model:

c_h = The holding cost per year
c_b = The backorder cost per unit backordered per cycle
c_o = The order cost per cycle
$C_o(Q) = c_oQ^\beta$ = The varying order cost per cycle, β is a real number
$\overline{D}$ = The average demand per year
$E(r-x) = ss$ = Safety stock = The expected net inventory
$\overline{H}$ = The average units on hand inventory $= \frac{\text{Max. on hand}+\text{Min. on hand}}{2} = \frac{ss+Q+ss}{2} = \frac{Q}{2} + ss = \frac{Q}{2} + r - E(x)$
K = The limitation on the expected annual holding cost
L = The lead time between the placement of an order and its receipt
n = The number of cycles
N = The inventory cycle
Q = The order quantity per cycle
x = The continuous random variable represents the units demanded during L
$f(x)$ = The probability density function of the lead-time demand x
$r-x$ = The random variable represents the net inventory
$p(x>r)$ = The probability of shortage $= \int_r^\infty f(x)\,dx = P(r) = 1 - F(r)$ = The reliability function
$\overline{B}(r)$ = The expected number of backorders per cycle $= \int_r^\infty (x-r)f(x)\,dx$

3. The mathematical model

Using the expression of the expected value of a random variable, it is possible to develop the expected annual total cost as follows:

$$E(\text{Total Cost}) = E(\text{Order Cost}) + E(\text{Holding Cost}) + E(\text{Backorders Cost}).$$
$$\text{i.e.,}\quad E(TC) = E(OC) + E(HC) + E(BC) \tag{1}$$

where

$$E(OC) = C_o(Q)\cdot n = c_oQ^\beta\frac{\overline{D}}{Q} = c_o\overline{D}Q^{\beta-1} \tag{2}$$

$$E(HC) = c_h\overline{H} = c_h\left(\frac{Q}{2} + r - E(x)\right) \tag{3}$$

and

$$E(BC) = c_b\cdot n\cdot \overline{B}(r) = \frac{c_b\overline{D}}{Q}\int_r^\infty (x-r)f(x)\,dx \tag{4}$$

Therefore

$$E[TC(Q,r)] = c_o\overline{D}Q^{\beta-1} + c_h\left(\frac{Q}{2} + r - E(x)\right) + \frac{c_b\overline{D}}{Q}\times\int_r^\infty (x-r)f(x)\,dx \tag{5}$$

Our objective is to minimize the expected annual total cost $E[TC(Q,r)]$ under the following constraint:

$$c_h\left(\frac{Q}{2} + r - E(x)\right) \leqslant K \tag{6}$$

To solve this primal function which is a convex programming problem, let us write it in the following form:

$$E[TC(Q,r)] = c_o\overline{D}Q^{\beta-1} + c_h\left(\frac{Q}{2} + r - E(x)\right) + \frac{c_b\overline{D}}{Q}\overline{B}(r) \tag{7}$$

$$\text{subject to}: \quad c_h\left(\frac{Q}{2} + r - E(x)\right) \leqslant K \tag{8}$$

To find the optimal values Q^* and r^* which minimize Eq. (7) under the constraint (8), we will use the Lagrangian multiplier technique as follows:

$$L(Q,r,\lambda) = c_o\overline{D}Q^{\beta-1} + c_h\left(\frac{Q}{2} + r - E(x)\right) + \frac{c_b\overline{D}}{Q}\overline{B}(r) + \lambda\left[c_h\left(\frac{Q}{2} + r - E(x)\right) - K\right] \tag{9}$$

where λ is the Lagrangian multiplier.

The optimal values Q^* and r^* can be found by setting each of the corresponding first partial derivatives of Eq. (9) equal to zero at $Q = Q^*$ and $r = r^*$ respectively, we obtain:

$$Q^{*2} + \frac{2(\beta-1)c_o\overline{D}}{(1+\lambda)c_h}Q^{*\beta} - \frac{2c_b\overline{D}}{(1+\lambda)c_h}\overline{B}(r) = 0 \tag{10}$$

and

$$P(r^*) = \frac{(1+\lambda)c_h}{c_b\overline{D}}Q^* \tag{11}$$

Clearly, it is difficult to find an exact solution of Q^* and r^* of Eqs. (10) and (11). So, we have to solve the two equations numerically, by the following algorithm that gives a closed approximate solution of these equations in a finite number of iterations:

- Step 1: Assume that $\overline{B} = 0$ and $r = E(x)$, then from Eq. (10) we have:

$$Q_1 = \left(\frac{2(1-\beta)c_o\overline{D}}{(1+\lambda)c_h}\right)^{\frac{1}{2-\beta}} \tag{12}$$

- Step 2: Substituting from Eq. (12) into Eq. (11) we get:

$$P(r_1) = \frac{(1+\lambda)c_h}{c_b\overline{D}}\left(\frac{2(1-\beta)c_o\overline{D}}{(1+\lambda)c_h}\right)^{\frac{1}{2-\beta}} \tag{13}$$

- Step 3: Substituting by r_1 from Eq. (13) into Eq. (10) to find Q_2 as:

$$Q_2^2 + \frac{2(\beta-1)c_o\overline{D}}{(1+\lambda)c_h}Q_2^\beta - \frac{2c_b\overline{D}}{(1+\lambda)c_h}\overline{B}(r_1) = 0 \tag{14}$$