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ScienceDirect
Int. J. Production Economics 108 (2007) 259–265
international journal of production economics
www.elsevier.com/locate/ijpe

Inventory model with stock-level dependent demand rate and variable holding cost

Hesham K. Alfares*

Systems Engineering Department, King Fahd University of Petroleum and Minerals, P.O. Box 5067, Dhahran 31261, Saudi Arabia

Available online 16 January 2007

Abstract

Inventory models in which the demand rate depends on the inventory level are based on the common real-life observation that greater product availability tends to stimulate more sales. Previous models incorporating inventory-level dependent demand rate assume that the holding cost is constant for the entire inventory cycle. This paper considers the inventory policy for an item with a stock-level dependent demand rate and a storage-time dependent holding cost. The holding cost per unit of the item per unit time is assumed to be an increasing function of the time spent in storage. Two time-dependent holding cost step functions are considered: Retroactive holding cost increase, and incremental holding cost increase. Procedures are developed for determining the optimal order quantity and the optimal cycle time for both cost structures.

Keywords: Inventory models; Stock-dependent demand; Variable holding cost; Optimization

1. Introduction

In traditional inventory models, the demand rate is assumed to be a given constant. Various inventory models have been developed for dealing with varying and stochastic demand. All these models implicitly assume that the demand rate is independent, i.e. an external parameter not influenced by the internal inventory policy. In real life, however, it is frequently observed that demand for a particular product can indeed be influenced by internal factors such as price and availability. The change in the demand in response to inventory or marketing decisions is commonly referred to as demand elasticity.

Most models that consider demand variation in response to item availability (i.e. inventory level) assume that the holding cost is constant for the entire inventory cycle. This paper presents an inventory model with a stock-level dependent demand rate and a variable holding cost. In this model, the holding cost is an increasing step function of the time spent in storage. Two types of time-dependent holding cost increase functions are considered: Retroactive increase, and incremental increase. For each type, a simple algorithm that minimizes the total inventory cost (TIC) is developed for calculating the optimal order quantity and associated cycle time.

As far as the author knows, the step structure of the holding cost function is unique to this paper. This structure is representative of many real-life situations in which the storage times can be

*Fax: +9663 860 2965.
E-mail address: hesham@ccse.kfupm.edu.sa.

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doi:10.1016/j.ijpe.2006.12.013

classified into different ranges, each with its distinctive unit holding cost. This is particularly true in the storage of deteriorating and perishable items such as food products. The longer these food products are kept in storage, the more sophisticated the storage facilities and services needed, and therefore, the higher the holding cost. For example, three different holding cost rates may apply to short-term, medium-term, and long-term food storage.

The remainder of this paper is organized as follows. Relevant literature is reviewed in the next section. This is followed by defining the problem and scope and developing the inventory model. Subsequently, the model is analyzed and a solution algorithm is presented for Case 1, retroactive holding cost increase. A similar analysis is followed for Case 2, incremental holding cost increase. Finally, suggestions and concluding remarks are given.

2. Literature review

Various models have been proposed for stock-level dependent inventory systems. Baker and Urban (1988a) investigated a deterministic inventory system in which the demand rate dependence on the inventory level is described by a polynomial function. A non-linear programming algorithm is utilized to determine the optimal order size and the reorder point. Urban (1995) investigated an inventory system in which the demand rate during stock-out periods differs from the in-stock period demand by a given amount. The demand rate depends on both the initial stock and the instantaneous stock. Urban formulates a profit-maximizing model and develops a closed-form solution.

A number of authors investigated inventory systems with a two-stage demand rate. Baker and Urban (1988b) considered an inventory system with an initial period of level-dependent demand followed by a period of constant demand. The analysis conducted on this model imposes a terminal condition of zero inventories at the end of the order cycle. Datta and Pal (1990) analyzed an infinite time horizon deterministic inventory system without shortage, which has a level-dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle. Paul et al. (1996) investigated a deterministic inventory system in which shortages are allowed and are fully backlogged. The demand is stock dependent to a certain level and then constant for the remaining periods. A flow chart is provided to solve the general system.

Pal et al. (1993) developed a deterministic inventory model assuming that the demand rate is stock dependent and that the items deteriorate at a constant rate θ. The net profit over one production run is maximized by numerically solving two nonlinear equations, and the optimal solution is compared with the no deterioration ($\theta = 0$) case. Hwang and Hahn (2000) constructed an inventory model for an item with an inventory-level dependent demand rate and a fixed expiry date. All units that are not sold by their expiry date are regarded as useless and therefore discarded. Separable programming is utilized to determine the optimal order level and order cycle length.

The holding cost is explicitly assumed to be varying over time in only few inventory models. Giri et al. (1996) developed a generalized EOQ model for deteriorating items with shortages, in which both the demand rate and the holding cost are continuous functions of time. The optimal inventory policy is derived assuming a finite planning horizon and constant replenishment cycles. Ray and Chaudhuri (1997) take the time value of money into account in analyzing an inventory system with stock-dependent demand rate and shortages. Two types of inflation rates are considered: internal (company) inflation, and external (general economy) inflation.

Shao et al. (2000) determined the optimum quality target for a manufacturing process where several grades of customer specifications may be sold. Since rejected goods could be stored and sold later to another customer, variable holding costs are considered in the model. Beltran and Krass (2002) analyzed the dynamic lot sizing problem with positive or negative demands and allowed disposal of excess inventory. Assuming deterministic time-varying demands and concave holding costs, an efficient dynamic programming algorithm is developed for this finite time horizon problem.

Goh (1994) apparently provides the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. Actually, Goh (1994) considers two types of holding cost variation: (a) a nonlinear function of storage time and (b) a nonlinear function of storage level. In this paper, we present a different functional form of the holding cost storage time dependence. While Goh (1992) models holding cost variation over time as a continuous nonlinear function, this paper