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A production–recycling–inventory system with imprecise holding costs

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Abstract

This paper presents an optimal control recycling production inventory system in fuzzy environment. The used items are bought back and then either put on recycling or disposal. Recycled products can be used for the new products which are sold again. Here, the rate of production, recycling and disposal are assumed to be function of time and considered as control variables. The demand inversely depends on the selling price. Again selling price is serviceable stock dependent. The holding costs (for serviceable and non-serviceable items) are fuzzy variables. At first we define the expected values of fuzzy variable, then the system is transferred to the fuzzy expected value model. In this paper, an optimal control approach is proposed to optimize the production, recycling and disposal strategy with respect so that expected value of total profit is maximum. The optimum results are presented both in tabular form and graphically.

Keywords: Production–recycling; Expected value of fuzzy variable; Credibility measure; Disposal; Inventory dependent selling price

1. Introduction

Production–recycling system is now-a-days an important area of inventory studies, due to growing environmental concern and environmental regulations in industry.

Fig. 3 is represented a simple production–recycling system. Some items are bought back from market for a recoverable inventory after being used by the customers. The serviceable stock is built up by both production and recycling and then delivered to the customer demand. The non-recycling items out of recovered products are disposal of. Thus two different stocks are maintained – one by fresh products from production and recycling system and other by recovered items purchased from market. The non-serviceable, i.e., recovered stock is supplied for either recycling or disposal. A number of research papers have already been published on the above type of models by Minner and Kleber [1], Dobos and Richter [2] and others.

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A product is promoted, now a days, through advertisements in modern electronic/mass media and/or by decorative and attractive display in showrooms. According to Levin et al. [3], "it is a common belief that large piles of goods displayed in a super market will lead the customers to buy more". Recent market research also recognizes this relationship. For this reason, several authors – (cf. [4–6], etc.) – presented some inventory models with stock-dependent demand. But for selecting an item for use, its selling price is one of the decisive factors in the present competitive market. It is a common practice that the lower selling price of an item causes a higher demand of that item whereas a higher selling price has the reverse effect. Incorporating the effects of selling price on demand, several researchers, e.g., [7] investigated the dependence on pricing.

Firstly Zadeh [8] developed possibility theory which was perfected and has became a strong tool to deal with in complete uncertain situation. Afterwards, Zimmermann [9], Dubois and Prade [10] employed the theory successfully to optimization problems.

In this paper is to represent fuzzy veritable expected value models (EVMS) (cf. [11]), where input data (serviceable and non-serviceable holding costs) are imprecise and decried by fuzzy variable. Here demand is selling price dependent. Also selling price is inventory levels dependent. Production, recycling and disposal consider as control variables. Production is serviceable stock dependent and unknown. The rate of production decreases as serviceable stock increases. The total profit is maximized formulating the problem as an optimal control problem. It is solved by the general expect value model, calculus method and generalized reduced gradient (GRG) technique (cf. Gabriel and Ragsdell [12]). The optimum production and stock levels are determined for known demand function. The model is illustrated through numerical examples and results are also presented graphically.

The paper is organized as follows. We present, in Section 2, basic fuzzy sets and fuzzy numbers, in Section 3, the assumptions and notations, in Section 4, the model formulation, in Section 5, solution methodology by using the calculus. Numerical experiment is performed and results are presented in Section 6. In Sections 7 and 8, some discussions and conclusions are made.

2. Basic fuzzy sets and fuzzy numbers

Fuzzy set: A Fuzzy set $\widetilde{A}$ in a universe of discourse X is defined as the following set of pairs

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : \ x \in X\},$$

where $\mu_{\widetilde{A}} : X \to [0,1]$ is a mapping called the membership function of the fuzzy set $\widetilde{A}$ and $\mu_{\widetilde{A}}(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set $\widetilde{A}$. The larger $\mu_{\widetilde{A}}(x)$ is the stronger grade of membership form in $\widetilde{A}$.

Convex fuzzy set: A fuzzy set $\widetilde{A}$ of the universe of discourse X is convex iff $\forall x_1, x_2 \in X$,

$$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geqslant \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)) \quad \text{when } 0 \leqslant \lambda \leqslant 1.$$

Normal fuzzy set: A fuzzy set $\widetilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that here exists at least one $x \in X$ such that $\mu_{\widetilde{A}}(x) = 1$.

A fuzzy number is a special case of a fuzzy set. Different definitions and properties of fuzzy numbers are encountered in the literature but they all agree on that a fuzzy number represents the conception of a set of real numbers close to 'a' where 'a' is the number being fuzzy field.

Triangular Fuzzy Number: Triangular Fuzzy Number (TFN) ($\tilde{a}$) (see Fig. 1) is the fuzzy number with the membership function $\mu_{\widetilde{A}}(x)$, a continuous mapping: $\mu_{\widetilde{A}}(x) : R \to [0,1]$

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{for} \quad -\infty < x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & \text{for} \quad a_1 \leqslant x < a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{for} \quad a_2 \leqslant x \leqslant a_3 \\ 0 & \text{for} \quad a_3 < x < \infty \end{cases}$$