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Dynamic pricing and advertising of perishable products with inventory holding costs

Rainer Schlosser

Hasso Plattner Institute, University of Potsdam, Germany

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A B S T R A C T

We examine a special class of dynamic pricing and advertising models for the sale of perishable goods, including marginal unit costs and inventory holding costs. The time horizon is assumed to be finite and we allow several model parameters to be dependent on time. For the stochastic version of the model, we derive closed-form expressions of the value function as well as of the optimal pricing and advertising policy in feedback form. Moreover, we show that for small unit shares, the model converges to a deterministic version of the problem, whose explicit solution is characterized by an overage and an underage case. We quantify the close relationship between the open-loop solution of the deterministic model and the expected evolution of optimally controlled stochastic sales processes. For both models, we derive sensitivity results. We find that in the case of positive holding costs, on average, optimal prices increase in time and advertising rates decrease. Furthermore, we analytically verify the excellent quality of optimal feedback policies of deterministic models applied in stochastic models.

1. Introduction

In a variety of practical applications, revenue management models are used for the sale of perishable products. Examples of such models can be found in a variety of contexts, for instance, fashion markets, fruit markets, ticket sales of special events, or transportation services. Managers dynamically set prices in order to maximize expected profits. Often sales costs (e.g., shipping costs) as well as inventory holding costs have to be taken into account (e.g., storage costs). Moreover, in some applications, advertising is also used to attract more customers. In this case, the seller is required to choose appropriate pricing and advertising decisions simultaneously. For this reason, a dynamic pricing model with a finite time horizon and endogenized advertising effects is needed in which the difference of expected revenues and expected expenditures is maximized.

In this paper, we consider a time-dependent stochastic dynamic pricing and advertising model for the sale of a stock of perishable products including marginal unit costs and inventory holding costs. Unfortunately, closed form solutions of such models barely exist. Hence, powerful heuristics as well as explicit special case solutions are very useful. Our aim is (i) to derive optimal policies, (ii) to evaluate optimal controlled sales processes, and (iii) to identify heuristics that are nearly optimal.

The best way to sell perishable products is a classical application of revenue management theory. The problem is closely related to the field of dynamic pricing which is summarized in the books by Talluri and van Ryzin (2004) and Phillips (2005).

E-mail address: rainer.schlosser@hpi.de

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The surveys by McGill and van Ryzin (1999), Bitran and Caldentey (2003), Elmaghraby and Keskinocak (2003), and Shen and Su (2007) and Goensch et al. (2009) provide an excellent overview of the details and the specific results that have been obtained over the last two decades.

The literature on dynamic pricing strategies that incorporate joint advertising effects is surprisingly very limited. After all, in many businesses, setting prices and determining the level of advertising expenditure are two of the key marketing-mix variables. In recent articles, mostly deterministic dynamic pricing and advertising models have been considered for the sale of durable goods. Particularly, the impact of discounting and specific adoption effects have been studied intensively in the case of isoelastic demand in an infinite horizon framework, for example, Sethi et al. (2008) and Helmes et al. (2013). Duopoly and oligopoly extension of these models are discussed in Krishnamoorthy et al. (2010) and Helmes and Schlosser (2015). A deterministic model that examines the sale of perishable products using joint pricing and advertising in the case of linear demand is presented in Feng et al. (2015). In their model inventory holding costs as well as an upper bound for the advertising rate are taken into account.

To our knowledge, there are only two papers that provide solutions of stochastic dynamic pricing and advertising models for the sale of perishable products: MacDonald and Rasmussen (2010) and Helmes and Schlosser (2013). In both models, neither marginal unit costs nor inventory holding costs are included. The latter article is based on the pure pricing model by McAfee and te Velde (2008) which is characterized by isoelastic demand and zero unit costs. The article by MacDonald and Rasmussen (2010) is related to the time-homogeneous pure pricing model by Gallego and van Ryzin (1994) with exponential demand. By considering stochastic and deterministic versions of their model, Gallego and van Ryzin (1994) show that in the case of a large inventory level and time independent demand, fixed price policy heuristics are asymptotically optimal. Based on their model, MacDonald and Rasmussen (2010) include isoelastic dynamic advertising effects and derive closed form solution formulas of the extended model. For their time-homogeneous stochastic model, they show that fixed price fixed advertising policies are a good heuristic as long as the initial inventory level is large.

In this paper, we analyze the relationship between stochastic and deterministic models in detail. Hence, we will solve both stochastic and deterministic versions of our model. Furthermore, instead of examining only large inventory levels, we consider the sale of shares of different sizes in order to determine the asymptotic behavior of stochastic model solutions for small unit shares. By evaluating the expected sales process of the stochastic model as well as the optimal sales path of the related deterministic model, we illustrate the close relationship between both versions of the model. We find that sensitivity results obtained for the deterministic model provide additional economic insight in deterministic and stochastic models. Moreover, we show that in our time-dependent model, the optimal controls can have complex evolutions. Hence, in general, simple policy heuristics that are constant in time will not be appropriate. Instead, we are motivated to apply optimal feedback policies of related deterministic models as a heuristic in stochastic models. Note, in general, the deterministic version of the considered kinds of models is easier to solve.

The main contributions of this paper are (i) a time-dependent dynamic pricing and advertising model with inventory holding costs is solved; (ii) optimal policies of stochastic and deterministic versions of the model are compared; (iii) optimally controlled sales processes are evaluated over time; and (iv) a general heuristic approach derived from the solution of the deterministic model is proposed.

This paper is organized as follows. In Section 2.1, we introduce the stochastic time-dependent pricing and advertising model with unit shares of size h. The model contains salvage values for unsold items as well as time-dependent inventory holding costs. The arrival rate of potential customers and the unit costs may also depend on time. In Section 2.2 we derive the solution of the model and give explicit formulas for the value function as well as the optimal feedback controls. In order to evaluate the optimal policy, we derive formulas for state probabilities of the optimally controlled inventory process and illustrate examples. In Section 2.3, we prove the convergence of the value function for small shares h to a two-part limit function.

In Section 3, we consider the deterministic version of the model with continuous state space. It is revealed that the limit function determined in Section 2.3 coincides with the solution of the Hamilton–Jacobi–Bellman (HJB) equation, which is associated with the deterministic control problem. The corresponding solution in feedback form is presented in Section 3.2. In Section 3.3, we determine the optimal inventory path and evaluate the optimally controlled sales process as a function of time, cf. open-loop form. Moreover, the results are compared to its stochastic counterparts. To identify characteristic properties of the complex interplay between various model parameters, sensitivity results are presented. Finally, in Section 4, we verify the excellent performance of heuristics that are based on the feedback solution of the deterministic model applied in a stochastic environment. We determine the expected profits of these suboptimal policies by solving the associated difference–differential equations explicitly. In Section 5, we summarize our results and give management recommendations. The proofs of our results are relegated to the Appendix.

2. Analytical solution of the stochastic model

2.1. Model description

We consider the situation where a monopolist wants to sell N items, $N<\infty$, of a perishable product over a finite time horizon T. We extend the time-homogeneous model by MacDonald and Rasmussen (2010) to a fairly general time inhomogeneous one including time-dependent unit costs and holding costs.