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Asset-selling problems with holding costs

Israel David*, Ofer Levi

Department of Industrial Engineering and Management, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

Abstract

Sequential stochastic assignment problems now comprise a significant literature that includes such important economical applications as the classical asset-selling problem and labor-market analysis (job search). In this type of problems there is a stream of bidders to whom several identical units at the disposal of the decision maker have to be sold. In this paper we incorporate holding costs to be incurred on the units (say assets) at hand into the classical model. Optimal strategies are defined as selling decision-rules which maximize the total expected net reward from the units.

We take advantage of the specific structure offered by the framework of sequential stochastic assignment to get explicit results for the optimal strategies. It is further shown how to implement these results for important specific bid distributions. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Inventory modeling; Sequential stochastic assignment; Asset-selling; Threshold policies; Dynamic programming; Exact results

1. Introduction

In this work we attempt to bridge a certain gap between two generic frameworks in the operations research literature, inventory theory and sequential stochastic assignment. The latter group of problems, abbreviated henceforth as SSAP's, deal with the following basic scenario: there are n identical units to be assigned, one at a time, to coming bidders. Bid values are independent, identically distributed, positive variables. The stream of bids constitutes a point process in time and when a bid arrives, a decision should be made whether to accept it or not. Rejected bids are no longer available. In addition, bid values are penalized by their arrival time. The penalizing mechanism (which becomes necessary in case of an unlimited stream of bidders) is often called a "discount function". The objective is to find an optimal assignment strategy that makes the appropriate tradeoff between accepting higher values and avoiding the penalty on delay.

Work on SSAP's originated from the seminal paper by Derman et al. [1] and now occupies a literature of its own (see [2–4] for citations and reviews, which also link the subject matter to literature on optimal stopping and "secretary" problems). Optimal decision rules for these models are usually characterized by a series of threshold functions $\{y_n(t)\}$ such that at time t, given n remaining units and a bid of value x pending, one accepts the bid if $x \geqslant y_n(t)$. If the nonincreasing discount function is $r(t)$, so that the discounted value of x accepted at t is $r(t)x$, and if $E_n(t)$ is (loosely speaking) the

* Corresponding author. Tel.: + 972-7-64-72-202; fax: + 972-7-64-72-958.

E-mail address: idavid@black.bgu.ac.il (I. David).

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value of the problem at t, then $E_n(t) = r(t)\sum_{i=1}^{n} y_i(t)$. This nice property holds under varied conditions (see e.g. [1,5]) and it bears a significant analytical and computational importance.

Traditionally, the main applications of SSAP's were generalizing the renowned Karlin's (single) asset-selling model [6]. Further applications were in the analysis of the labor market (Lippman and McCall [7]), and in the study of economics models of price-search [8]. Several further applications have been assigned to these models – see e.g. Righter [3] and David [2] for engineering applications. Notably, SSAP's fit into *yield management* which recently gained considerable interest with regard to the airline and hotel industries and to freight transport. See the Encyclopedia [9] for a short description of this area and Weatherford and Bodily [10] for a taxonomy and research overview till 1992. A valuable recent book on yield management is [11].

In this preliminary paper we take the generic model of SSAP, or multi-asset selling problem, and incorporate the *holding costs* incurred on unsold assets. Thus we do not assume any discount function, a deadline for the process or any restrictions on bid arrival – the incentive to make do with lower bids would lie in the accumulated holding costs. (These costs replace the SSAP-typical discount function in a non-trivial way). The objective is to maximize the total expected *net* profit from the units at hand, that is to say the prices accepted for them minus the holding costs. Thus, the effect of time is also conveyed exclusively by the holding costs.

Our models apply primarily to the classical economical interpretation of the model as describing asset selling processes. As in many of similar prototypic models, we assume the stationarity of monetary conditions and arrival rates. We consequently find that in the case of fixed-rate holding costs and *general* renewal arrival of i.i.d. bidders with no recall, the optimal decision rules come out nicely and their characteristic threshold values may be given explicit solution. We demonstrate our results with concrete examples.

Section 2 below specifies the model and the necessary notation. Section 3 contains the main results, namely expressing the value of the problem in terms of the pertinent threshold values and providing a key scalar equation for each threshold value. Several consequences are also mentioned. In Section 4 we implement the main results for the cases of uniform, normal and finitely discrete bid-value distributions. Section 5 is the conclusion.

2. The model

We consider a supplier or holder of identical items who faces a potentially infinite stream of bidders. Bidders are customers who arrive sequentially, each of whom offers a random price he or she is willing to pay for a single unit. One unit is sold at a time and rejected bids cannot be accepted later. The bid arrivals constitute a *general* renewal process, for which the mean interarrival time is denoted by τ. $0 < \tau < \infty$. Bids are positive, independent, identically distributed random variables. The random variable X denotes the bid-value and F denotes the cumulative probability distribution function of X.

Holding costs are charged against the remaining units at a fixed rate h – the seller pays h monetary units per item held per unit-time. We denote by E_n the supremal total net profit expected from n remaining units (revenue from these units minus future holding costs), starting when a bid is pending – just before its value is revealed – till the end of the process when the inventory level hits zero.

It is obvious that if a bid of value x is accepted, and $y > x$, then a bid of value y is also accepted. Conversely, if a bid of value r is rejected and $s < r$ then a bid of value s is also rejected. It follows that the optimal selling rule is of a *threshold* type. The threshold value for the random bid given an inventory of size n will be denoted by y_n.

By its definition, and by assuming a renewal process with i.i.d. bid-value, E_n is time-independent. The threshold value series defines the optimal policy (the policy which maximizes E_n) in the following way: given n assets (or units) at hand, the decision maker should accept a bid of value x if and only if $x \geqslant y_n$ (in fact for $x = y_n$ accepting and rejecting the bid are equally profitable in expectation). Since E_n does not vary with time, and since y_n is a unique