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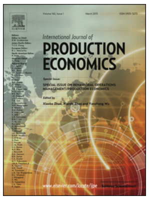

Avoiding the bullwhip effect using Damped Trend forecasting and the Order-Up-To replenishment policy

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ABSTRACT

We study the Damped Trend forecasting method and its bullwhip generating behaviour when used within the Order-Up-To (OUT) replenishment policy. Using *z*-transform transfer functions we determine complete stability criteria for the Damped Trend forecasting method. We show that this forecasting mechanism is stable for a much larger proportion of the parametrical space than is generally acknowledged in the literature. We provide a new proof to the known fact that the Naïve, Exponential Smoothing and Holts Method forecasting, when used inside the OUT policy, will always generate bullwhip for every possible demand process, for any lead-time. Further, we demonstrate the Damped Trend OUT system behaves differently. Sometimes it will generate bullwhip and sometimes it will not. Bullwhip avoidance occurs when demand is dominated by low frequency harmonics in some instances. In other instances bullwhip avoidance happens when demand is dominated by high frequency harmonics. We derive sufficient conditions for when bullwhip will definitely be generated and necessary conditions for when bullwhip may be avoided. We verify our analytical findings with a numerical investigation.

1. Introduction

The Damped Trend (DT) forecasting method generalises the Holts model for forecasting a linear trend by adding a damping parameter ϕ to the trend component. Although such exponential smoothing systems with damping parameters had been noted earlier (see for instance, Gardner (1985), Gilchrist (1976), and Roberts (1982)), Gardner and McKenzie (1985) were the first to present both a theoretical and an empirical investigation of the system. Since then, it has often been promoted as the most accurate forecasting technique in the so-called M-competitions (Makridakis and Hibon, 2000). Gardner and McKenzie (2011) find the DT method is the best method for 84% of the 3003 time series in the M3 forecasting competition when using local initial values. It was the best method 70% of the time when using global initial values. Fildes et al. (2008) concluded that DT forecasting can “reasonably claim to be a benchmark forecasting method for all others to beat”. The great virtual of DT is that future forecasts are not simply flat line extensions of the current, next period forecast. It is able to detect and forecast both linear and exponential trends. The DT forecasting methodology also contains at least 11 different forecasting methods when all of the three parameters are selected from the real [0,1] interval (Gardner and McKenzie, 2011). This makes it a powerful and very general forecasting approach for short term demand data as tuning the DT parameters effectively automates model selection. In this paper, as they are industrially popular, we are particularly interested in studying the performance of Naïve, Simple Exponential Smoothing, Holts Method and Damped Trend forecasting procedures.

A series of papers (Gardner, 1985, 1990; Gardner and McKenzie, 1985, 1989) have proposed restricting the damping parameter to the [0,1] interval, and that the rate of decay $(1-\phi)$ increases with the noise in the series because the difference between a damped and a linear trend can be substantial over long time horizons. When $\phi>1$, both Gardner (1985) and Gardner and McKenzie (1985) explain that the forecast exhibits an exponential growth over time and is probably a dangerous option to use in an automatic forecasting procedure. However, both Tashman and Kruk (1996) and Taylor (2003) argue that there can be value in allowing $\phi>1$ as it could be suited to time series with strong increasing trends. Roberts (1982) mentions that a negative ϕ is possible, but generally negative damping parameters are not discussed in the literature. In this paper, we will investigate the general DT method without any restrictions on the range of values any parameter can take.

Although DT method is claimed to be superior to many other methods from a forecasting perspective, only a few studies are available that demonstrate the managerial importance of DT forecasting in the supply chain, inventory or operations management

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fields. Snyder et al. (2002) studies the exponential smoothing family of forecasting methods (including Damped Trend) in the Order-Up-To inventory control policy. They use a bootstrap method to determine the total lead-time demand distribution and measure performance via the fill rate. Acar and Gardner (2012) investigate the use of the DT method in a real supply chain based on the trade-offs recommended by Gardner (1990) and show that the DT method outperforms Simple Exponential Smoothing and the Holts method.

We study forecasting techniques from a supply chain perspective, using the forecasts inside the Order-Up-To (OUT) replenishment policy and evaluating performance via the bullwhip criterion. This focus allows us to sharpen the results of Dejonckheere et al. (2003) for the Naïve and the Simple Exponential Smoothing forecasting techniques and to introduce new results for the Holts and Damped Trend forecasting methods.

Our specific focus is to evaluate the use of the DT forecast for use within the linear OUT replenishment policy (Chen et al., 2000). This industrially popular policy has been selected for our study as it is the optimal linear policy for minimising local inventory costs and it operates on the same discrete time periodic basis as the DT forecasting method. The OUT policy requires two forecasts, one forecast of demand over the lead-time, and one forecast of demand in the period after the lead-time. The OUT policy is a popular rule as, for any set of forecasts the policy determines replenishment quantities that minimise the variance of the inventory levels, Vassian (1955) and Hosoda and Disney (2006).

Note also that the OUT policy is not restricted to certain demand patterns. This suggests the OUT policy would be suitable when demand contains significant trends. The minimised inventory variance leads to reduced safety stock requirements in supply chains (and lower inventory costs) in order to meet availability targets. However, most OUT policy settings result in a phenomenon where the variance of the orders is greater than the variance of the demand, a.k.a. the bullwhip effect. This bullwhip effect is costly in supply chains as it creates costs either in the form of labour idling and over-time or excessive and unused capacity. It has been shown that the bullwhip effect is related to the variance of forecasts as well as the variance of the forecast errors of demand over the lead-time and review period, Chen et al. (2000), Dejonckheere et al. (2003).

In this paper, since the combined DT/OUT system consists of six linear discrete time difference equations, the z-transform and Fourier transform approaches are used for the analysis. z-Transform approaches have a long history in production control and inventory management. Vassian (1955) appears to be the first to apply the z-transform to an inventory control problem. Brown (1963) is perhaps the first textbook that details how to apply z-transform techniques to forecasting problems. Adelson (1966) studies the dynamic behaviour of coupled forecasting and scheduling systems. More recently, Popplewell and Bonney (1987) study the structure of MRP systems with z-transforms and Hoberg, Thonemann and Bradley (2007) use z-transforms to study bullwhip behaviour in linear supply chains.

The use of the Fourier Transform for studying the dynamic behaviour of forecasting and replenishment systems is less common: Dejonckheere et al. (2003) use the Fourier transform to investigate the behaviour of the OUT policy with exponential smoothing and moving average forecasts. Ouyang and Daganzo (2006) characterise the bullwhip effect in linear supply chains with Robust Control techniques. The frequency response approach is particularly potent as it is able to generate results that are applicable for any demand process. This is because all demand processes can be decomposed into a set of harmonic frequencies via the Fourier transform. By understanding how the system reacts to the complete set of harmonic frequencies (via the amplitude ratio within the frequency response plot, a.k.a. the Bode plot) we are able to gain insights that are valid for all possible demand patterns. Many of the results that we obtain are also valid for any lead-time.

Section 2 briefly introduces the DT forecasting mechanism and derives its discrete time transfer function. We then study the stability boundaries of DT forecasting mechanism in Section 3. In Section 4, we incorporate the DT forecasting methodology into the OUT replenishment policy, and develop a discrete-time z-transform transfer function representation of the combined forecasting and replenishment system. Then, in Section 5, we analyse the frequency response plot in order to ascertain the bullwhip performance of the system. Section 6 provides numerical simulation results confirming our theoretical findings. Section 7 concludes.

2. The Damped Trend forecasting method

There appears to be no commonly accepted set of notation for the Damped Trend forecasting method. Different authors use different notation and several different formulations of the DT model exist. We prefer to use the recursion form due to Gardner and McKenzie (1985) shown in (1) but have made slight changes to notation in order to avoid $\{S,T,L\}$ which already have implied meanings in the supply chain management literature. This three dimensional form of the DT model is the most general form in the literature.

$$\left.\begin{aligned}\hat{a}_t &= (1-\alpha)(\hat{a}_{t-1}+\phi\hat{b}_{t-1})+\alpha d_t\\ \hat{b}_t &= (1-\beta)\phi\hat{b}_{t-1}+\beta(\hat{a}_t-\hat{a}_{t-1})\\ \hat{d}_{t,t+k} &= \hat{a}_t+\hat{b}_t\gamma(k,\phi)\end{aligned}\right\}. \tag{1}$$

In the linear discrete time difference equations of (1), d_t is the time series being forecasted, in our context we refer to it as “demand at time t”. $\hat{a}_t$ is the current estimate of the level, exponentially smoothed by the constant level α. $\hat{b}_t$ is the current estimate of the trend, exponentially smoothed by the constant β. $\{\alpha,\beta\}$ are the “smoothing parameters”. ϕ is the damping parameter that can be interpreted as a measure of the persistence of the trend. k is the number of periods ahead that the forecast is required to predict. $\hat{d}_{t,t+k}$ is the forecast, made at time t, of the demand in the period $t+k$ and $\gamma(k,\phi)=\sum_{i=1}^{k}\phi^i=\phi(\phi^k-1)/(\phi-1)$. The first equation of (1) suggests that the estimated demand consists of a time dependant “level” component, the second equations tracks a trend component and the third equation combines the level and trend estimates to make a forecast k periods ahead. The behaviour of $\gamma(k,\phi)$ is quite rich. When $\phi>1$ then $\gamma(k,\phi)$ exhibits positive exponential growth over k. $\phi=1$ implies a $\gamma(k,\phi)$ with positive linear growth over k. $0<\phi<1$ produces a positive damped exponential growth that approaches $\phi/(1-\phi)$ as $k\to\infty$. When $\phi=0$ then $\gamma(k,\phi)=0$. When $-1<\phi<0$, $\gamma(k,\phi)$ has a two period oscillation that is always negative and converges to $\phi/(1-\phi)$. $\phi<-1$ results in a $\gamma(k,\phi)$ with a two period oscillation that alternates between positive and negative numbers with ever increasing amplitude over k.

Several well-known forecasting approaches are encapsulated within the DT model (Gardner and McKenzie, 2011). These include the Holts method when $\phi=1$ where there is no damping of the trend component, Simple Exponential Smoothing (SES) when $\beta=0$ and $\phi=0$, and Naïve forecasting when $\alpha=1$, $\beta=0$ and $\phi=0$, see Table 1. We study the system performance when the OUT policy exploits either DT forecasting, or these special cases, to predict demand over the lead-time and review period.

One of our main methodological tools is transfer functions. Transfer functions are useful for studying linear dynamic systems, as they allow convolution in the time domain to be replaced by simple algebra in the complex frequency domain. In the frequency domain there is also a wide range of tools developed by control