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RESEARCH PAPER

Bullwhip Effect Analysis in Supply Chain for Demand Forecasting Technology

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Abstract: On the basis of the AR(1) stochastic process model for consumer demand which was introduced by Professor H.L.Lee, qualified and simulation model of bullwhip effect are established when order-up-to inventory policy is employed, which investigate demand variability caused by forecasting technology, such as moving average (MA) method, exponentially weighted moving average (EWMA) method or mean square error-optimal (MSE-optimal) forecasting method. The influence of forecasting methods on bullwhip effect are also analyzed, and carried out rules decreasing the bullwhip effect for employing MA, EWMA or MSE-optimal forecasting scheme.

Key Words: supply chain; bullwhip effect; moving average forecasting method; exponential weighted moving average method; mean square error-optimal forecasting method

1 Introduction

An important observation in supply chain management, known as the bullwhip effect, suggests that demand variability increases as one moves up a supply chain. It can misguide capacity plans and miss production schedules without being able to see the sales of its products at the distribution channel stage when a supply chain is plagued with a bullwhip effect. It also leads to many inefficiencies: insufficient or excessive capacities, excessive inventory investment, poor customer service as a result of unavailable products or long backlogs, lost revenues, uncertain production planning (i.e., excessive revisions), and high costs for corrections.

The bullwhip effect is not a recent observation. In the early 1960s Professor Forrest[1] first analyzed the system characteristic that demand information amplified as it is transmitted up the chain using system dynamics, and illustrates its existence with a series of case studies. In an inventory management experimental context in MIT, Professor Sterman[2] also reports evidence of the bullwhip effect in the "Beer Distribution Game." Sterman[2] interprets the phenomenon as a consequence of players' systematic irrational behavior, or "misperceptions of feedback." In the early 1990s, the bullwhip phenomenon has been recognized in many companies, such as Procter & Gamble, General Electric Co., Ford, Chrysler, Hewlett-Packard, Compaq, where their product sales and the distributors' orders were investigated.

The bullwhip effect was systemically analyzed by Professor Lee[3−4] and his cooperators. They established a bullwhip effect theory model that was used to discover the course of the bullwhip phenomenon. They claimed that demand distortion may arise as a result of optimizing behaviors by players in the supply chain and identified four main causes of the bullwhip effect: forecasting technology for demand, batch ordering, price variations and rationing game between retailers as a result of supply shortages. The recent researches on the bullwhip effect are spread out around the upper frame of reference[5−9].

This article focuses on the impact of difference demand forecasting technology on the bullwhip effect. On the basis of the bullwhip effect theory model of simple two-level supply chain with AR(1) stationary demand process presented by Lee, an extensive work is performed systematically with respect to the influences on different information forecasting which mainly includes, establishing the two-level supply chain bullwhip effect theory models and simulation models on the basis of AR(1) stationary demand process, in which the moving average (MA) forecasting method, exponential weighted moving average (EWMA) method, mean square error-optimal (MSE-optimal) forecasting method, and order-up-to inventory policy are employed. The influence on the bullwhip effect is compared with three forecasting methods and some references and guides are provided for the decrease of the supply chain bullwhip effect.

2 Problem definition

Consider a simple two-stage supply chain consisting of a single retailer and a single manufacturer. We suppose that the retailer is the exclusive buyer from the manufacturer, and only one product is exchanged between them. The action occurs in an infinity discrete time t, which is $(-\infty, \dots, -1, 0, 1, \dots, \infty)$.

At the end of period t, the retailer estimates the demand

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during t+1 periods following certain forecasting scheme and order-up-to inventory policy according to the observed demand, and places an order, q_t, to the manufacturer at the beginning of period t+1. Furthermore, assuming the lead time is l, the retailer will receive the product at the start of period $t+l$+1.

Suppose the customer demand, D_t, seen by the retailer is an AR(1) demand process given as follows:

$$D_t = \mu + \rho D_{t-1} + \varepsilon_t \tag{1}$$

where μ is a nonnegative constant, ρ is a correlation parameter between the neighboring period demands with $|\rho| < 1$, and the error terms, ε_t, which are independent and identically distributed (i.i.d.) from a symmetric distribution with mean 0 and variance σ^2. Note that if $\rho = 0$, Equation (1) implies that the demands D_t are i.i.d. with mean μ and variance σ^2.

We can easily obtain the following equation from Equation (1):

$$\begin{cases} E(D_t) = \dfrac{\mu}{1-\rho} \\ Var(D_t) = \dfrac{\sigma^2}{1-\rho^2} \end{cases} \tag{2}$$

We assume that the retailer uses simple MA, EWMA, or MSE-optimal forecasting method and order-up-to inventory policy to estimate $\hat{D}_t^l$, which is the total expected demand over l periods after period t, then S_t is estimated from the observed demand as Equation (3), and the order q_t to the manufacturer will be as Equation (4):

$$S_t = \hat{D}_t^l + z\hat{\sigma}_t^l \tag{3}$$

$$q_t = S_t - S_{t-1} + D_{t-1} \tag{4}$$

where $z\hat{\sigma}_t^l$ is an estimate security inventory in the l period, z is a constant chosen to meet a desired service level, and $\hat{\sigma}_t^l$ is an estimate of the standard deviation of the l period forecast error. Our objective is to quantify the bullwhip effect. To do this, we must determine the variance of q_t relative to the variance of D_t , apparently, the quantified bullwhip effect of this two-level supply chain is $Var(q_t)/Var(D_t)$.

3 Bullwhip effect under MA

Suppose that the retailer uses a simple MA forecasting method to estimate $\hat{D}_t^l$ and $\hat{\sigma}_t^l$ on the basis of the demand observations from the previous p periods. That is,

$$\hat{D}_t^l = l\left(\frac{\sum\limits_{i=1}^{p} D_{t-i}}{p}\right) \tag{5}$$

We can write the order quantity q_t placed by the retailer to the manufacture from Equations (3) and (4) as

$$\begin{aligned} q_t &= \hat{D}_t^l - \hat{D}_{t-1}^l + z(\hat{\sigma}_t^l - \hat{\sigma}_{t-1}^l) + D_t \\ &= l\left(\frac{D_{t-1} - D_{t-p-1}}{p}\right) + D_{t-1} - z(\hat{\sigma}_t^l - \hat{\sigma}_{t-1}^l) \\ &= \left(1 + \frac{l}{p}\right) D_{t-1} - \left(\frac{l}{p}\right) D_{t-p-1} + z(\hat{\sigma}_t^l - \hat{\sigma}_{t-1}^l) \end{aligned} \tag{6}$$

In order to simplify analysis, we assume further that $z = 0$, that is, the security inventory is taken no account of, so the variance of q_t is given as follows:

$$\begin{aligned} Var(q_t) = &\left(1 + \frac{l}{p}\right)^2 Var(D_{t-1}) - 2\left(\frac{l}{p}\right)\left(1 + \frac{l}{p}\right) \\ &Cov(D_{t-1}, D_{t-p-1}) + \left(\frac{l}{p}\right)^2 Var(D_{t-p-1}) \\ = &\left(1 + \frac{2l}{p} + \frac{2l^2}{p^2}\right) Var(D) - \left(\frac{2l}{p} + \frac{2l^2}{p^2}\right) \rho^p Var(D) \\ = &\left[1 + \left(\frac{2l}{p} + \frac{2l^2}{p^2}\right)(1 - \rho^p)\right] Var(D) \end{aligned} \tag{7}$$

where the upper equation follows from,

$$\begin{aligned} &Cov(D_{t-1}, D_{t-p-1}) = \frac{\sigma^2 \rho^p}{1-\rho^2} \\ &Var(D) = Var(D_t) = Var(D_{t-1}) = \cdots = \frac{\sigma^2}{1-\rho^2} \end{aligned} \tag{8}$$

Thus we have the following variability from the retailer to the manufacturer while $z = 0$:

$$\frac{Var(q)}{Var(D)} = 1 + (\frac{2l}{p} + \frac{2l^2}{p^2})(1 - \rho^p) > 1 \tag{9}$$

Equation (9) shows that the variance of the order, or upstream demand process, is greater than the variance of the demand process, the bullwhip effect exists, when order-up-to policy and the MA forecasting method are employed.

4 Bullwhip effect under EWMA

Under the same policy and demand process scenario, when the EWMA forecasting method is employed, the estimate of the total expected demand over l periods after period t, $\hat{D}_t^l$, is given by

$$\hat{D}_t^l = l[\alpha D_t + (1-\alpha)\hat{D}_{t-1}] \tag{10}$$

where $\alpha (0 < \alpha < 1)$ is the smoothing constant. Similar to the MA technique, forecasts for periods $t + i$ $(i = 1, 2, \dots)$ made at time t are equal. Replacing (10) in (3) and (4), we find that q_t is as follow:

$$\begin{aligned} q_t &= \hat{D}_t^l - \hat{D}_{t-1}^l + z(\hat{\sigma}_t^l - \hat{\sigma}_{t-1}^l) + D_t \\ &= (1 + \alpha l)D_{t-1} - \alpha l\hat{D}_{t-1} + z(\hat{\sigma}_t^l - \hat{\sigma}_{t-1}^l) \end{aligned} \tag{11}$$

Similarly, we suppose that the retailer ignores the security inventory. The variance of q_t is found as follows:

$$\begin{aligned} Var(q_t) = &(1 + \alpha l)^2 Var(D_{t-1}) - 2\alpha l(1 + \alpha l) \\ &Cov(D_{t-1}, \hat{D}_{t-1}) + \alpha^2 l^2 Var(\hat{D}_{t-1}) \\ = &\left[1 + \frac{2\alpha l[2 - (1-L)\alpha]}{1 - (1-\alpha)\rho} \cdot \frac{1-\rho}{2-\alpha}\right] Var(D) \end{aligned} \tag{12}$$

where

$$\begin{aligned} Cov(D_{t-1}, \hat{D}_{t-1}) &= Cov\left[D_{t-1}, \sum_{i=1}^{t-2} \alpha(1-\alpha)^{i-1} D_{t-1-i}\right] \\ &= \frac{\alpha\rho}{1-(1-\alpha)\rho} \cdot \frac{\sigma^2}{1-\rho^2} \end{aligned} \tag{13}$$