



# The analysis of bullwhip effect in a HMMS-type supply chain

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## ABSTRACT

The aim of the paper is to investigate the well-known bullwhip effect of supply chains. Control theoretic analysis of bullwhip effect is extensively analyzed in the literature with the Laplace transform. This paper tries to examine the effect for an extended Holt–Modigliani–Muth–Simon model. A two-stage supply chain (supplier–manufacturer) is studied with quadratic costs functional. It is assumed that both firms minimize the relevant costs. The order of the manufacturer is delayed with a known constant. Two cases are examined: supplier and manufacturer minimize the relevant costs decentralized, and a centralized decision rule. The question is answered, how to decrease the bullwhip effect.

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## 1. Introduction

Bullwhip effect refers to the connections in supply chains. This effect explains the fluctuations of sales (demand), manufacturing, and supply. It was first observed by Forrester (1961) when studying industrial dynamics. Lee et al. (1997) have newly discovered this phenomenon. They have mentioned four basic causes of bullwhip effect:

- Forrester effect, or lead-times and demand signal processing,
- Burbidge effect, or order batching,
- Houlihan effect, or rationing and gaming, and
- promotion effect, or price fluctuations.

These new names after known scientists have been given by Disney and Towill (2003).

Modeling of bullwhip effect was first made by Metters (1997). He has developed a stochastic model to discuss the fluctuations. Other stochastic modeling of the effect was fulfilled by, e.g. Cachon (1999), Kelle and Milne (1999), Chen et al. (2000), Machuca and Barajas (2004), and Pujawan (2004) in an EOQ-environment. Chen et al. (2000) have explicitly proven that variation ratio of demand and manufacturing is strictly greater, than one, i.e. the fluctuations are increasing along the supply chains. These investigations examine the Forrester and Burbidge effects. A new direction in analysis of bullwhip effect is to apply fuzzy logic instead of stochastic modeling. (Carlsson and Fullér, 2000, 2001) Another generalization of the analysis of bullwhip effect is to investigate this phenomenon in closed-loop supply chain, or in reverse logistics environment, see Zhou et al. (2006), and Zhou and Disney (2006).

Deterministic and control-theoretic investigations of bullwhip effect are made by Dejonckheere et al. (2003), Disney and Towill (2003), Geary et al. (2006), Towill et al. (2007) and Zhou et al. (2010).

In this paper we will examine the bullwhip effect in a HMMS-type supply chain. The bullwhip effect in an HMMS-type environment was *not* investigated before, so we focus on the analysis of this model. Holt et al. (1960) have developed a quadratic production planning model which was tested in a paint factory. In the last years the HMMS-model is newly analyzed extensively in the literature. (Dobos, 2003; Singhal and Singhal, 2007; Atici and Uysal, 2008).

The aim of the paper is to investigate a HMMS-type (Holt et al., 1960) two-stage supply chain, and to analyze whether the bullwhip effect appears in this model. To show the bullwhip effect, we develop two models: a decentralized and a centralized HMMS-type supply chain models. The decentralized model assumes that the manufacturer solves his production planning problem, and then the ordering process is based on this optimal production plan. Then the supplier minimizes the costs on the basis of the ordering of the supplier. In the centralized model it is assumed that the participants of the supply chain cooperate, i.e. they minimize the sum of the costs of manufacturer and supplier. Then we compare the production-inventory strategies of the manufacturer and supplier to answer the question, whether the bullwhip effect will be reduced with the cooperation. This cooperation can be defined, as a kind of information sharing between the parties of the supply chain. The model is a continuous time control theoretic model with quadratic cost functional.

The paper organized as follows. The decentralized model is presented in Section 2. In this section we pose the used model and we solve it. The next section discusses the centralized supply chain model with its solution. Sections 2 and 3 contain numerical examples to demonstrate the results. In Section 4 we compare the optimal solutions of Sections 2 and 3, and another example is provided to show the concept of cooperation between the participants of the supply chain. Last we summarize the results of the paper.

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**2. Basic model: decentralized system**

We examine a simple supply chain. The supply chain contains two firms, a supplier and a manufacturer. We assume that the firms are independent, i.e. they make decision to minimize their own costs. The firms have two stores: a store for raw materials and a store for end products. We will assume that the input stores are empty, i.e. the firms can order suitable quantity, and they get the ordered quantity. The production processes have a known, constant lead time. The material flow of the model is depicted in Fig. 1.

The following parameters are used in the models:

- $T$  length of the planning horizon,
- $S(t)$  the rate of demand, continuous differentiable,  $t \in [0, T]$ ,
- $\tau_m$  lead time of manufacturing process,
- $\tau_s$  lead time of supply process,
- $\bar{I}_m(t)$  inventory goal size of manufactured product,  $t \in [0, T]$ ,
- $\bar{I}_s(t)$  inventory goal size of supplied product,  $t \in [-\tau_m, T - \tau_m]$ ,
- $\bar{P}_m(t)$  manufacturing goal level,  $t \in [0, T]$ ,
- $\bar{P}_s(t)$  supply goal level,  $t \in [-\tau_m, T - \tau_m]$ ,
- $h_m$  inventory holding cost coefficient in manufactured product store,
- $h_s$  inventory holding cost coefficient in supplied product store,
- $c_m$  production cost coefficient for manufacturing,
- $c_s$  production cost coefficient for supply.

In the HMMS-model it is assumed that the management of the (manufacturer and supplier) firms have fixed a production-inventory pattern, i.e. the production plans  $\bar{P}_m(t)$  and  $\bar{P}_s(t)$ , and planned inventory levels  $\bar{I}_m(t)$  and  $\bar{I}_s(t)$  are known before the planning horizon. The goal of the managers of the firms is to minimize the deviations from the fixed goal level, i.e. plans. The deviations are defined, as quadratic functionals with known parameters. This phenomenon was empirically tested by Holt, Modigliani, Holt, and Simon in 1960.

Decision variables:

- $I_m(t)$  inventory level of manufactured product, non-negative,  $t \in [0, T]$ ,
- $I_s(t)$  inventory level of supplied product, non-negative,  $t \in [-\tau_m, T - \tau_m]$ ,
- $\tilde{I}_s(t)$  new inventory level of supplied product, non-negative,  $t \in [0, T]$ ,
- $P_m(t)$  rate of manufacturing, non-negative,  $t \in [0, T]$ ,
- $P_s(t)$  rate of supply, non-negative,  $t \in [-\tau_m, T - \tau_m]$ ,
- $\tilde{P}_s(t)$  new rate of supply, non-negative,  $t \in [0, T]$ .

The decentralized model examines a situation when the supplier and the manufacturer decide independently. It means that the manufacturer determines its optimal production-inventory strategy then it orders the necessary quantity of products to meet the known demand. The supplier accepts the order, and it minimizes the relevant costs. The cost function of supplier and manufacturer consists of two parts: quadratic production and inventory costs.

Let us now model the manufacturer in this HMMS-environment. (See Holt-Modigliani-Muth-Simon (1960)) The first decision is made by

the manufacturer, and then the supplier determines the optimal production policy. The manufacturer will minimize the quadratic costs, as follows:

$$\dot{I}_m(t) = P_m(t) - S(t), \quad I_m(0) = I_{m0}, \quad 0 \leq t \leq T, \tag{1}$$

$$\int_0^T \left\{ \frac{h_m}{2} [I_m(t) - \bar{I}_m(t)]^2 + \frac{c_m}{2} [P_m(t) - \bar{P}_m(t)]^2 \right\} dt \rightarrow \min. \tag{2}$$

Let us assume that the optimal production-inventory policy of manufacturer is  $(I_m^d(t), P_m^d(t))$  in models (1) and (2). The manufacturer orders  $P^d(t + \tau_m)$  to meet the manufacturing needs. The supplier must solve the next problem (3) and (4) to satisfy the order of manufacturer:

$$\dot{I}_s(t) = P_s(t) - P_m^d(t + \tau_m), \quad I_s(-\tau_m) = I_{s0}, \quad -\tau_m \leq t \leq T - \tau_m, \tag{3}$$

$$\int_{-\tau_m}^{T - \tau_m} \left\{ \frac{h_s}{2} [I_s(t) - \bar{I}_s(t)]^2 + \frac{c_s}{2} [P_s(t) - \bar{P}_s(t)]^2 \right\} dt \rightarrow \min. \tag{4}$$

Let us now transform the stock-flow equation of supplier in time:

$$\dot{\tilde{I}}_s(t) = \tilde{P}_s(t) - P_m(t), \quad \tilde{I}_s(0) = I_s(-\tau_m), \quad 0 \leq t \leq T, \tag{5}$$

and goal functional

$$\int_0^T \left\{ \frac{h_s}{2} [\tilde{I}_s(t) - \bar{I}_s(t - \tau_m)]^2 + \frac{c_s}{2} [\tilde{P}_s(t) - \bar{P}_s(t - \tau_m)]^2 \right\} dt, \tag{6}$$

where the new decision variables are

$$\tilde{I}_s(t) = I_s(t - \tau_m), \quad \tilde{P}_s(t) = P_s(t - \tau_m).$$

We will solve the problem (5) and (6) instead models (3) and (4). The problem has the same planning horizon  $[0, T]$ , as models (1) and (2).

**Lemma 1.** Let us assume that production-inventory strategy  $(I_m^d(t), P_m^d(t))$  is an optimal solution for models (1) and (2). Then the optimal solution must satisfy the following differential equation:

$$\begin{pmatrix} \dot{I}_m^d(t) \\ \dot{P}_m^d(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{h_m}{c_m} & 0 \end{pmatrix} \begin{pmatrix} I_m^d(t) \\ P_m^d(t) \end{pmatrix} + \begin{pmatrix} -S(t) \\ \dot{P}_m(t) - \frac{h_m}{c_m} \dot{I}_m(t) \end{pmatrix},$$

with initial and ending values

$$I_m^d(0) = I_{m0}, \quad P_m^d(T) = \bar{P}_m(T).$$

We will not prove the lemma, the proof can be found in paper Dobos (2003). If production strategy  $P_m^d(t)$  is known, than problems (5) and (6) can be solved.

**Lemma 2.** Let us assume that production-inventory strategy  $(\tilde{I}_s^d(t), \tilde{P}_s^d(t))$  is an optimal solution for models (5) and (6). Then the optimal solution must satisfy the following differential equation:

$$\begin{pmatrix} \dot{\tilde{I}}_s^d(t) \\ \dot{\tilde{P}}_s^d(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{h_s}{c_s} & 0 \end{pmatrix} \begin{pmatrix} \tilde{I}_s^d(t) \\ \tilde{P}_s^d(t) \end{pmatrix} + \begin{pmatrix} -P_m^d(t) \\ \dot{\tilde{P}}_s(t) - \frac{h_s}{c_s} \dot{\tilde{I}}_s(t) \end{pmatrix},$$

with initial and ending values

$$\tilde{I}_s^d(0) = I_{s0}, \quad \tilde{P}_s^d(T) = \bar{P}_s(T).$$

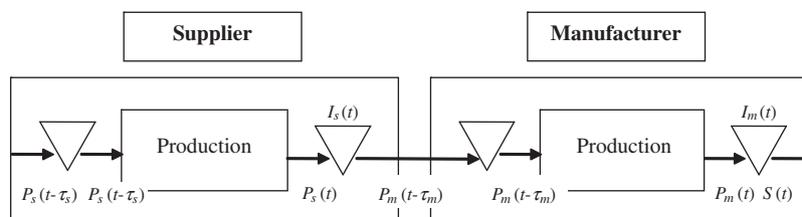


Fig. 1. Material flow in the models.

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