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Quantitative techniques for financial risk assessment: a comparative approach using different risk measures and estimation methods

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Abstract

The aim of this paper is to highlight and illustrate the use of some quantitative techniques for risk estimation in finance and insurance. The first component involved in risk assessment concerns the risk measure used and the second one is based on the estimation technique. We will study the theoretical properties, the accuracy of modeling the economic phenomena and the computational performances of the risk measures Value-at-Risk, Conditional Tail Expectation, Conditional Value-at-Risk and Limited Value-at-Risk in the case of logistic distribution. We also investigate the most important statistical estimation methods for risk measure evaluation and we will compare their theoretical and empirical behavior. The quality of the risk estimation process corresponding to the quantitative techniques discussed will be tested for both real and simulated data. Numerical results will be provided.

1. Introduction

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 The increasing complexity of the problems arising in various fields determined a strong demand for efficient methods of decision making. At the same time, the progress of information technology determined the development of advanced computational techniques to implement such methods. Risk assessment provides the theoretical basis for decision making processes in finance and insurance. Risk management has received a considerable interest among researchers in the last years. An important problem for portfolio managers, investors and financial regulators, refers to risk modeling and estimation. In his paper, Markowitz, 1959 underlined that investors should take into account not only the expected return, but also the variance of the return and to choose the portfolio with the highest expected return for a given level of the variance. Generally, the mean–variance analysis is applied when the returns are assumed to be normally distributed or when the investor's preferences can be accurately described using the mean and the variance. Recently, (Fulga and Dedu, 2010, Dedu and Fulga, 2011, Tudor, 2012, Toma, 2012, Toma and Leoni-Aubin, 2013), proposed new risk measures and optimization techniques to addressing various limitations of the mean–variance approach. In this paper we study some quantitative techniques for risk modeling and estimation in finance and insurance. In Section 2, we study theoretical properties, the accuracy of modeling the economic phenomena and the computational performance of different risk measures, such as: Value-at-Risk, Conditional Tail Expectation, Conditional Value-at-Risk and Limited Value-at-Risk. In Section 3, the most important Value-at-Risk estimation techniques are presented. In Section 4, we derive analytical formulas for Value-at-Risk and Conditional Value-at-Risk in the case when the loss random variable follows a Logistic distribution. Section 5 provides computational results of the case study. Section 6 presents the conclusions.

2. Risk measures used in the finance and insurance

2.1. Value-at-Risk

 In the class of quantile-based risk measures, the most used is Value-at-Risk, which evaluates the maximal loss that can occur in a time horizon with a given probability level. Let $X : \Omega \to \mathbf{R}$ a random variable defined on the probability space (Ω, K, P), with cumulative distribution function $F_X(x) = P(X \leq x), \forall x \in \mathbf{R}$.

 Let $\alpha \in (0, 1)$. The Value-at-Risk corresponding to a random variable X at the probability level α is given by:

$$\text{VaR}_\alpha(X) = \inf\{x \in \mathbf{R} \mid P(X \leq x) \geq \alpha\}.$$

 If the random variable X has a continuous one-to-one cumulative distribution function, then $\text{VaR}_\alpha(X)$ can be computed as the unique solution of the equation:

$$\text{VaR}_\alpha(X) = F_X^{-1}(\alpha). \qquad (1)$$

 Value-at-Risk is used for setting the capital adequacy limits for banks and other financial institutions and plays an important role in investment, risk management and regulatory control of financial institutions.

2.2. Conditional Tail Expectation

 The limitations of the most common used risk measures (like variance, which is a symmetric measure, or Value-at-Risk, which does not take into account the extreme values) and the criticism addressed to some of these measures (Artzner, 1999), led to the development of new risk measures, with good analytical properties.

 The Conditional Tail Expectation of the random variable X at the probability level α is defined by:

$$\text{CTE}_\alpha(X) = \text{E}\big[X \mid X \geq \text{VaR}_\alpha(X)\big].$$

2.3. Conditional Value-at-Risk