Journal of Econometrics 187 (2015) 472–485

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

A stochastic dominance approach to financial risk management strategies[☆]

Chia-Lin Chang [a,b], Juan-Ángel Jiménez-Martín [c], Esfandiar Maasoumi [d,*], Teodosio Pérez-Amaral [c]

[a] Department of Applied Economics, National Chung Hsing University, Taichung, Taiwan
[b] Department of Finance, National Chung Hsing University, Taichung, Taiwan
[c] Department of Quantitative Economics, Complutense University of Madrid, Spain
[d] Department of Economics, Emory University, USA

ARTICLE INFO

Article history:
Available online 10 March 2015

JEL classification:
G32
G11
G17
C53
C22

Keywords:
Stochastic dominance
Value-at-Risk
Daily capital charges
Violation penalties
Optimizing strategy
Basel III Accord
VIX futures
Global financial crisis

ABSTRACT

The Basel III Accord requires that banks and other Authorized Deposit-taking Institutions (ADIs) communicate their daily risk forecasts to the appropriate monetary authorities at the beginning of each trading day, using one of a range of alternative risk models to forecast Value-at-Risk (VaR). The risk estimates from these models are used to determine the daily capital charges (DCC) and associated capital costs of ADIs, depending in part on the number of previous violations, whereby realized losses exceed the estimated VaR. In this paper we define risk management in terms of choosing sensibly from a variety of risk models and discuss the optimal selection of the risk models. Previous approaches to model selection for predicting VaR proposed combining alternative risk models and ranking such models on the basis of *average* DCC, or other quantiles of its distribution. These methods are based on the first moment, or specific quantiles of the DCC distribution, and supported by restrictive evaluation functions. In this paper, we consider robust uniform rankings of models over large classes of loss functions that may reflect different weights and concerns over different intervals of the distribution of losses and DCC. The uniform rankings are based on recently developed statistical tests of stochastic dominance (SD). The SD tests are illustrated using the prices and returns of VIX futures. The empirical findings show that the tests of SD can rank different pairs of models to a statistical degree of confidence, and that the alternative (recentered) SD tests are in general agreement.

1. Introduction

The Basel III Accord requires that banks and other Authorized Deposit-taking Institutions (ADIs) communicate their daily risk forecasts to the appropriate monetary authorities at the beginning of each trading day, using one of a range of alternative financial risk models to forecast Value-at-Risk (VaR). The risk estimates from these models are used to determine the daily capital charges (DCC) and associated capital costs of ADIs, depending in part on the number of previous violations, whereby realized losses exceed the estimated VaR (for further details see, for example, Chang et al., 2011).

In 1993 the Chicago Board Options Exchange (CBOE) introduced a volatility index, VIX (Whaley, 1993), which was originally designed to measure the market expectation of 30-day volatility implied by at-the-money S&P100 option prices. In 2003, together with Goldman Sachs, CBOE updated VIX to reflect a new way of measuring expected volatility, one that continues to be widely used by financial theorists.

The new VIX is based on the S&P500 Index, and estimates expected volatility by averaging the weighted prices of S&P500 puts and calls over a wide range of strike prices. Although many market participants considered the index to be a good predictor of short-term volatility, namely daily or intraday, it took several years for the market to introduce volatility products, starting with over-the-counter products, such as variance swaps and other financial derivatives. The first exchange-traded product, VIX futures, was

☆ The authors are most grateful to Michael McAleer for numerous insightful comments and guidance, and to two reviewers for very helpful comments and suggestions. For financial support, the first author wishes to thank the National Science Council, Taiwan, and the second and fourth authors acknowledge the Ministerio de Economía y Competitividad and Comunidad de Madrid, Spain.

* Corresponding author.
E-mail address: esfandiar.maasoumi@emory.edu (E. Maasoumi).

http://dx.doi.org/10.1016/j.jeconom.2015.02.032
0304-4076/

introduced in March 2004, and was followed by VIX options in February 2006. Both of these volatility derivatives are based on the VIX index as the underlying asset.

McAleer et al. (2013a,b,c) analyze, from a practical perspective, how the new market risk management strategies performed during the 2008–09 global financial crisis (GFC), and evaluate how the GFC affected the best risk management practices. These papers define risk management in terms of choosing appropriate financial targets, from a variety of financial risk models, and discuss the selection of optimal risk models. They forecast VaR using ten univariate conditional volatility models with different error distributions. Additionally, they analyze twelve new strategies based on combinations of the previous standard univariate model forecasts of VaR, namely: infimum (0th percentile), supremum (100th percentile), average, median and nine additional strategies based on the 10th through to the 90th percentiles. Such an approach is intended to select a robust VaR forecast, irrespective of the time period, that provides reasonable daily capital charges and number of violation penalties under the Basel II Accord. They found that the median is a GFC-robust strategy, in the sense that maintaining the same risk management strategy before, during and after the GFC leads to comparatively low daily capital charges and violation penalties under the Basel II Accord. Chang et al. (2011) apply a similar methodology for choosing the best strategy to forecast VaR for a portfolio based on VIX futures.

These prior methods focus on the first moment, or certain quantiles of the DCC distribution. Alternative criteria may consider mean–variance trade-offs, as in substantial areas of financial research, or general evaluation criteria that incorporate higher moments and quantiles of the underlying probability distributions. These will all provide appropriate "cardinal" and "complete" rankings of models and strategies, based on subjective valuations of different aspects, or parts of the DCC distribution. For instance, were DCC to be a Gaussian variate, mean–variance assessments would be strongly justified. This is not likely, however, and consensus on appropriate weighting and assessment functionals of non-Gaussian distributions has been elusive. It is of some importance to point out, that mean–variance type assessments are justified by a joint consideration of quadratic risk function, as well as the full characterization of the Gaussian case by the second moment. Absent a Gaussian setting, justification of a quadratic loss function itself becomes questionable. Why would we not be concerned with higher moments (when they exist), and often asymmetrical tail area behavior, especially when tail functions such as VaR are of interest?

A complementary robust alternative, is to seek weak uniform rankings over entire classes of evaluation functions, and based on nonparametric distributions of DCC. In this respect, Stochastic Dominance (SD) tests have been developed to test for statistically significant rankings of prospects. Assuming that F and G are the distribution functions of DCC produced by model 1 and model 2, respectively, model 1 first-order SD model 2, over the support of DCC, iff $F(\mathrm{DCC}) \leq G(\mathrm{DCC})$, with strict inequality over some values of DCC. This means that the model that produces G is dominant over all merely increasing evaluation functions since, at all quantiles, the probability that capital charges are smaller under G is greater than under F. In particular, the distribution F will have a higher median DCC than G. Similarly, each and every (quantile) percentile of the F distribution will be at a higher DCC level than the corresponding percentile of the G distribution. Consequently, model 2 will be preferred to model 1, to a statistical degree of confidence, on the basis of lower capital charges. Higher-order SD rankings reference further subclasses of evaluation functions, those that are increasing and concave, reflecting increasing risk aversion (see Sections 4–5 below).

In this paper we examine several standard models for forecasting VaRs, including GARCH, EGARCH, and, GJR, paired with

Table 1
Basel accord penalty zones.

Zone	Number of violations	k
Green	0 to 4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10+	1.00

Note: the number of violations is given for 250 business days. The penalty structure under the Basel II Accord is specified for the number of violations and not their magnitude, either individually or cumulatively.

Gaussian and Student-t distributions. The results show that the Gaussian distribution is preferred to Student-t in forecasting DCC. With the Student-t distribution, the EGARCH model provides a greater likelihood of higher DCC in comparison with GARCH and GJR. Using the Gaussian distribution to forecast DCC does not lead to either first- or second-order stochastic dominance. In respect of the CDF and integrated CDF, the basis of first- and second-order stochastic dominance testing, it seems that the higher expected DCC of GJR or GARCH may be compensated by lower risk compared with EGARCH.

The remainder of the paper is organized as follows. Section 2 describes briefly the Basel II Accord for computing daily capital charges. Section 3 presents alternative GARCH models to produce daily capital charges. In Section 4 the definition, notation and properties of stochastic dominance are presented. Section 5 introduces the data, describes the block bootstrapping method to simulate time series, and illustrates the application of stochastic dominance to enhance financial risk management strategies of banks. Section 6 presents the main results. Section 7 gives some concluding comments.

2. Forecasting value-at-risk and daily capital charges

In this section, which follows McAleer et al. (2013a,b,c) closely, we introduce the calculation of daily capital charges (DCC) as a basic criterion for choosing between risk models. The Basel II Accord stipulates that daily capital charges (DCC) must be set at the higher of the previous day's VaR or the average VaR over the last 60 business days, multiplied by a factor $(3 + k)$ for a violation penalty, where a violation occurs when the actual negative returns exceed the VaR forecast negative returns for a given day:

$$\mathrm{DCC}_t = \sup\left\{-(3+k)\,\overline{\mathrm{VaR}}_{60},\ \ -\mathrm{VaR}_{t-1}\right\} \tag{1}$$

where

$\mathrm{DCC}_t =$ daily capital charges,
$\mathrm{VaR}_t = \hat{Y}_t - z_t \cdot \hat{\sigma}_t$, the Value-at-Risk for day t,
$\overline{\mathrm{VaR}}_{60} =$ mean VaR over the previous 60 working days,
$\hat{Y}_t =$ estimated return at time t,
$z_t =$ 1% critical value of the distribution of returns at time t,
$\hat{\sigma}_t =$ estimated risk (or square root of volatility) at time t,
$0 \leq k \leq 1$ is the Basel II violation penalty (see Table 1).

It is well known that the formula given in Eq. (1) is contained in the 1995 amendment to Basel I, while Table 1 appears for the first time in the Basel II Accord in 2004. The multiplication factor (or penalty), k, depends on the central authority's assessment of the ADI's risk management practices and the results of a simple backtest. It is determined by the number of times actual losses exceed a particular day's VaR forecast (see Basel Committee on Banking Supervision (1As stated in a number of previous papers (see, for example, McAleer et al., 2013a,b,c), the minimum multiplication factor of 3 is intended to compensate for various errors that can arise in model implementation, such as simplifying