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Analysis of mean-VaR model for financial risk control

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Abstract

Financial risk control is a kind of complicated system engineering. This paper studies validity of portfolio investment of the mean-VaR model under holding period condition. The model is analyzed through Lagrange multiplier method, and the portfolio weight of global minimum VaR is also given by the portfolio weight combined of minimum variance and maximum Sharpe ratio.

Keywords: Financial engineering; portfolio selection; mean-VaR model; Lagrange multiplier; Sharpe ratio

1. Introduction

The classical mean-variance (M-V) model was first introduced by Markowitz [1] in 1952, which was generally considered as the foundation of the modern portfolio theory. In this model the risk of a portfolio was measured by the variance of the rate of return. In Shen [2], it analyzed the solution for the mean-variance model through the matrix approach, and gave some economic connotations for the portfolio.

With the progress of research, many scholars found that the variance was not a good measurement for the risk, so many new methods for measuring risk have been developed. Recently, Value at risk (VaR) which was accepted as a kind of important tools of financial risk control has been widely used in risk measurement and control field (see [3-5]). Jacson et al. [6] mentioned the Basel Accord which required that the financial institutions should use VaR to measure and disclosure the market risk. Xue et al. [7] studied three calculated methods of the given portfolio's VaR with different horizon and confidence level. Then the research of mean-VaR model became popular. The efficient frontier of mean-VaR model was discussed under the confidence level in [8]. A new mean-VaR model under holding period condition has been presented, and the portfolio with global minimum VaR for this model was also analyzed [9].

This paper gives the further studies on the mean-VaR model with holding period condition through the specific portfolios and discusses the relationship between the portfolio weight of global minimum VaR and two specific portfolio weights.

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2. Mean-VaR (M-VaR) Model

2.1. Mean-variance (M-V) model

Assume that there exist n assets and the return vectors of these assets have multivariate normal distribution. The expected return rate vector is $\boldsymbol{\mu}=(\mu_1, \mu_2,\ldots, \mu_n)^T$, then the mean variance model can be expressed as follows,

$$\begin{cases} \min \quad \sigma^2 = \boldsymbol{\omega}_{MV}^T \boldsymbol{\Omega} \boldsymbol{\omega}_{MV} \\ \text{s.t.} \quad \boldsymbol{\omega}_{MV}^T \boldsymbol{\mu} = \mu_p \\ \qquad \boldsymbol{\omega}_{MV}^T \mathbf{t} = 1 \end{cases} \tag{1}$$

where $\boldsymbol{\omega}_{MV}=(\omega_1, \omega_2,\ldots, \omega_n)^T$ denotes the portfolio weight vector, $\boldsymbol{\Omega}=(\sigma_{ij})_{n\times n}$ the variance covariance matrix, $\mathbf{t}=(1, 1, \ldots, 1)^T$ the unit vector and μ_p the expected portfolio return rate. The model can be solved by Lagrange multiplier,

$$\boldsymbol{\omega}_{MV} = \frac{B(A\mu_p - B)}{D}\frac{\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}}{B} + \frac{A(C - B\mu_p)}{D}\frac{\boldsymbol{\Omega}^{-1}\mathbf{t}}{A} \tag{2}$$

where $A = \mathbf{t}^T\boldsymbol{\Omega}^{-1}\mathbf{t}$, $B = \mathbf{t}^T\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}$, $C = \boldsymbol{\mu}^T\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}$, $D = AC - B^2$. Let

$$\boldsymbol{\omega}_\mu = \frac{\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}}{B}, \boldsymbol{\omega}_t = \frac{\boldsymbol{\Omega}^{-1}\mathbf{t}}{A}, a_\mu = \frac{B(A\mu_p - B)}{D}, a_t = \frac{A(C - B\mu_p)}{D}$$

Then the equation (2) can be written briefly as $\boldsymbol{\omega}_{MV} = a_\mu\boldsymbol{\omega}_\mu + a_t\boldsymbol{\omega}_t$. Here, $\boldsymbol{\omega}_\mu, \boldsymbol{\omega}_t$ are two important efficient portfolios given by Lemma 2.1.

Lemma 2.1 $\boldsymbol{\omega}_t$ is a portfolio on efficient frontier with minimum variance, and the mean and variance are $\mu_p=B/A$, $\sigma^2=1/A$, respectively; $\boldsymbol{\omega}_\mu$ is a portfolio on efficient frontier with maximum earning per risk (Sharpe ratio), and the mean and variance are $\mu_p=C/B$, $\sigma^2=C/B^2$, respectively.

As a kind of financial risk measurement, VaR is used to generalize the M-V model.

2.2. Mean-VaR model analysis

Let Z_α be the α-quantile of standard normal distribution, $\alpha\in(0.5, 1]$, and Δt be the holding period. Then the portfolio's VaR can be written in the following,

$$VaR = Z_\alpha \sigma \sqrt{\Delta t} - \mu_p \Delta t = Z_\alpha \sqrt{\boldsymbol{\omega}^T \boldsymbol{\Omega} \boldsymbol{\omega}} \sqrt{\Delta t} - \mu_p \Delta t \tag{3}$$

So the mean-VaR model can be given immediately (Hu et al. [9]),

$$\begin{cases} \min \quad VaR = Z_\alpha \sqrt{\boldsymbol{\omega}^T \boldsymbol{\Omega} \boldsymbol{\omega}} \sqrt{\Delta t} - \mu_p \Delta t \\ \text{s.t.} \quad \boldsymbol{\omega}^T \boldsymbol{\mu} = r_p \\ \qquad \boldsymbol{\omega}^T \mathbf{t} = 1 \end{cases} \tag{4}$$

Theorem 2.2 Let Z_α be the α-quantile of standard normal distribution, Δt be the holding period and μ_p be the expected portfolio return rate. The portfolio weight $\boldsymbol{\omega}$ on mean-VaR frontier should also be on mean-variance frontier.

Proof. The model (4) can be solved by Lagrange multiplier, let

$$F(\boldsymbol{\omega}, \lambda) = Z_\alpha \sqrt{\boldsymbol{\omega}^T \boldsymbol{\Omega} \boldsymbol{\omega}} \sqrt{\Delta t} - \mu_p \Delta t - \lambda_1(\boldsymbol{\omega}^T \boldsymbol{\mu} - \mu_p) - \lambda_2(\boldsymbol{\omega}^T \mathbf{t} - 1) \tag{5}$$