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Extremal financial risk models and portfolio evaluation

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Abstract

It is difficult to find an existing single model which is able to simultaneously model exceedances over thresholds in multivariate financial time series. A new modeling approach, which is a combination of max-stable processes, GARCH processes, and Markov processes, is proposed. Combining Markov processes and max-stable processes defines a new statistical model which has the flexibility of modeling cross-sectional tail dependencies between risk factors and tail dependencies across time. The new model also models asymmetric behaviors of negative and positive returns on financial assets. An important application of the proposed method is to calculate value at risk (VaR) and evaluate portfolio combinations under VaR constraints. Result comparisons between VaRs based on the new approach and VaRs based on some existing methods such as variance–covariance approach and historical simulation approach suggest that some existing methods substantially underestimate the risks during recession and expansion time.

Keywords: Extreme value theory; M4 processes; Markov chains; Tail dependence index; Financial risk; Portfolio evaluation

1. Extreme value approaches

The natural way to model extremal observations is to apply extreme value theory—for example, see discussions in Embrechts et al. (1997), Diebold et al. (1998), Neftci (2000), and Zhang (2005) for univariate cases, and Tawn (1990), Smith and Weissman (1996), Stărică (1999, 2000), Embrechts et al. (2000), Bouye (2002), Buhl et al. (2002), Mashal and Zeevi (2002), Smith (2003), Heffernan and Tawn (2004), and Zhang and Smith (2004b) for multivariate cases, among many others. Meanwhile, manageable models for joint cross-sectional and temporal extremal-dependent observations are still sparse, and applications of joint extreme financial models to risk assessments and portfolio evaluations during recession and expansion time are still a difficult task.

Perhaps, the best known model for cross-sectional tail dependence and sequential tail dependence is Smith and Weissman's (1996) multivariate maxima of moving maxima (henceforth M4) process model. Due to the degenerate properties of the joint distributions of the M4 process, it is not an easy task to estimate the parameter values and fit the model to data. The model was not popularized. Only recently, Zhang (2002, 2003, 2005), Zhang and Smith (2004a,b), and Chamú Morales (2005) systematically started building rigorous estimation theory of M4 processes and applications. In this paper, we further develop models for financial assets based on observed data dependence structures.

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The statistical evidences of tail dependence, also known as extremal dependence or asymptotic dependence, among financial asset returns have been shown by Embrechts et al. (2002), Zhang and Smith (2004b), Zhang (2005), and others. For the univariate case, Zhang (2005) proposes time series models to characterize nontransient jumps in returns (large observations). Here transience is interpreted as the situation that a large jump in returns does not have an (extreme) impact on the returns in the future. In this paper, we extend Zhang's univariate time series models to new multivariate time series models which include extreme dependence and extreme impact dependence structures. For univariate models dealing with the case of transient jumps in returns, we refer to those studied by Eraker et al. (2003), Duan et al. (2003).

Key to the construction of a multivariate time series model is the cross-sectional dependence structure among the variables. In financial time series modeling, it is well known that negative and positive returns are asymmetric, therefore it is preferred to employ different models to these two different kinds of returns. But in order to make statistical inferences, we need a combined model to model both returns simultaneously. In this paper, we build such models based on the dependence nature of three financial index return data sets. We propose a new model structure which we call it MCM4 (Markov chain M4) processes and show MCM4 processes are useful in modeling extreme observations simultaneously. Compared with models studied in Zhang and Smith (2004b), the new model is improved by introducing a three-state Markov chain which indicates financial returns are positive, negative, or no change. We also show that the new model structures reduce the complexity of parameter estimation as well.

Like other time series models, M4 process models have dimension indexes—i.e., the dimensions of the parameter space. However, unlike usual model selection procedures—such as AIC, BIC, PACF, ACF used in linear time series, we perform the gamma test (Zhang, 2004a) based on the exceedance values over a typical high threshold value to determine the dimensions of the parameter space before an M4 process model is fitted to data.

The definition of M4 processes will be stated first in Section 2. Then some basic properties will be studied. In Section 3 we briefly review the gamma test. Real-data analysis will be conducted in Sections 4 and 5. In Section 6, we compare value at risk (VaR) and evaluate simulated portfolio combinations. Discussions will be presented in Section 7. Proofs are deferred to the Appendix.

2. M4 processes and related properties

Smith and Weissman (1996), Zhang and Smith (2004a,b), and Zhang (2005) demonstrate that M4 processes have the ability to model both cross-sectional tail dependence and temporal tail dependence. A D-dimensional M4 process can be written as

$$Y_{id} = \max_{1\leqslant l\leqslant\infty} \max_{-\infty\leqslant k\leqslant\infty} a_{l,k,d} Z_{l,i-k}, \quad d=1,\ldots,D, \quad -\infty<i<\infty, \tag{1}$$

where Z_{lk} are independent unit Fréchet random variables which have distribution $F(x)=\mathrm{e}^{-1/x}$, $x>0$, nonnegative constant moving coefficients a_{lkd} satisfy $\sum_{l=1}^{\infty}\sum_{k=-\infty}^{\infty} a_{l,k,d}=1$, see Smith and Weissman (1996). For a particular d and a fixed k, $a_{l,k,d}$ define a moving pattern which is called a signature pattern,—see details in Zhang and Smith (2004a).

For this process, we have

$$\begin{aligned}
&P\{Y_{id}\leqslant y_{id},\ 1\leqslant i\leqslant r,\ 1\leqslant d\leqslant D\}\\
&\quad= P\left\{Z_{l,i-k}\leqslant \frac{y_{id}}{a_{l,k,d}} \text{ for } l\geqslant 1, -\infty<k<\infty, 1\leqslant i\leqslant r, 1\leqslant d\leqslant D\right\}\\
&\quad= P\left\{Z_{l,m}\leqslant \min_{1-m\leqslant k\leqslant r-m}\ \min_{1\leqslant d\leqslant D} \frac{y_{m+k,d}}{a_{l,k,d}}, l\geqslant 1, -\infty<m<\infty\right\}\\
&\quad= \exp\left[-\sum_{l=1}^{\infty}\sum_{m=-\infty}^{\infty} \max_{1-m\leqslant k\leqslant r-m}\ \max_{1\leqslant d\leqslant D} \frac{a_{l,k,d}}{y_{m+k,d}}\right].
\end{aligned} \tag{2}$$

Hence

$$P^n\{Y_{id}\leqslant ny_{id},\ 1\leqslant i\leqslant r,\ 1\leqslant d\leqslant D\} = P\{Y_{id}\leqslant y_{id}, 1\leqslant i\leqslant r, 1\leqslant d\leqslant D\}.$$

A process satisfying these conditions for all $r\geqslant 1$ is called max-stable (de Haan, 1984).