Insurance: Mathematics and Economics 57 (2014) 31–45

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Optimal risk and liquidity management with costly refinancing opportunities

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ARTICLE INFO

Article history:
Received April 2014
Received in revised form
May 2014
Accepted 5 May 2014

MSC:
91G10
91G50
91G80

Keywords:
Liquidity management
Risk exposure
Singular stochastic control
Stochastic impulse control

ABSTRACT

In this paper we study risk and liquidity management decisions within an insurance firm. Risk management corresponds to decisions regarding proportional reinsurance, whereas liquidity management has two components: distribution of dividends and costly equity issuance. Contingent on whether proportional or fixed costs of reinvestment are considered, singular stochastic control or stochastic impulse control techniques are used to seek strategies that maximize the firm value. We find that, in a proportional-costs setting, the optimal strategies are always mixed in terms of risk management and refinancing. In contrast, when fixed issuance costs are too high relative to the firm's profitability, optimal management does not involve refinancing. We provide analytical specifications of the optimal strategies, as well as a qualitative analysis of the interaction between refinancing and risk management.

1. Introduction

The aim of this work is to study the optimal decisions of an insurance firm's manager in terms of liquidity management and risk exposure. With respect to the latter, the manager continuously chooses the proportion of claims that are to be reinsured. This determines how risky/profitable the firm is at each date. Liquidity management entails dividend distribution in bonanza times and injections of fresh cash when required.

Risk management plays an essential role in the daily operations of an insurance firm and has a stark influence on the firm value. It is therefore not surprising that this aspect of the firm's management has been extensively studied. Liquidity management, and more importantly, how it feeds back to optimal risk management is also crucial. In good times, it provides shareholders with returns on their investment, whereas in bad times it may lend a necessary lifeline to the firm. Indeed, distressed equity issuances are not rare. Jostarndt reports in Jostarndt (2009) that in Germany, between 1996 and 2004, 123 out of 267 financially-troubled corporations issued new equity. Franks and Sanzhar document in Franks and Sanzhar (2005) that distressed equity issuances were a significant proportion of total seasoned issuances in the United Kingdom from 1989 to 1998. In our model, the firm requires a positive level of cash reserves to fund its day-to-day operations. Should the said level become non-positive, the firm must be either liquidated or recapitalized, which is interpreted as costly equity issuance. We consider both the cases of proportional and fixed issuance costs, and study the interaction between risk-management and liquidity decisions both analytically and numerically.

In contrast with the classical Merton problem, in our model shareholders are risk neutral. This requires different mathematical techniques than in the classical case, namely singular stochastic control.[1] To the best of our knowledge, Jeanblanc-Picqué and Shiryaev (1995) and Radner and Shepp (1996) were the first ones to work in such a setting. In their cases, the risky project has a fixed size, which would amount to no reinsurance, and no reinvestment is possible. The manager then faces the compromise of distributing too much of the firm's cash reserves as dividends, which is costly due to bankruptcy risk, or postponing distribution too long. The latter option prolongs the firm's lifespan, but it is inefficient

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[1] Technically, risk-neutrality implies that the set of feasible liquidity strategies cannot be assumed to consist exclusively of absolutely continuous processes.

http://dx.doi.org/10.1016/j.insmatheco.2014.05.001
0167-6687/

because of discounting. The authors show that the optimal strategy is as follows: postpone distributing dividends as long as the cash reserves remain under a certain threshold $x^* \in \mathbb{R}_+$; whenever the current reserves level x is greater than x^*, distribute $x - x^*$ immediately. Since the level of cash reserves is a continuous process, this results in a localized optimal strategy. The solution to the manager's problem may be, therefore, identified with a Skorokhod problem, where the cash-reserves process is reflected at the level x^*. The mathematical methodologies for these kinds of problems, which broadly speaking may be called instantaneous control of stochastic processes, have been studied, for example, by Bass and Hsu (1990) and by Harrison and Taksar (1983).

Within the insurance literature, Højgaard and Taksar pioneered in Højgaard and Taksar (1998) the use of an instantaneously-controlled Brownian motion to model the net earnings of an insurance firm. We use their model as a benchmark. Here the manager controls the firm's risk exposure via proportional reinsurance, represented by the control variable $1 - \alpha \in [0, 1]$. The second control variable, the cumulative distribution of dividends, keeps the firm's cash reserves below or at the optimal dividend-distribution barrier x^*. The possibility of continuously fine-tuning the reinsurance level results in an additional (relative to Jeanblanc-Picqué and Shiryaev, 1995) boundary point $0 < x^{**} < x^*$. Below x^{**} there is partial reinsurance and on (x^{**}, x^*) there is none. In the partial-reinsurance region, the optimal choice α^* results in a reserves level that follows a geometric Brownian motion: the proportion of reinsurance tends to one as the reserves approach zero, thus preventing the firm from ever going bankrupt. In other words, in the world of Højgaard and Taksar a firm never defaults strategically. This model could also be interpreted as a Merton-style problem of optimal portfolio design for a risk-neutral investor, where α represents the proportion of wealth invested in the risky asset, and where the savings account pays zero interest.

We extend the model in Højgaard and Taksar (1998) to allow for costly equity issuance as a means to refinance the firm. We find that, in the proportional-costs case, the manager never forgoes the refinancing option and, interestingly, never chooses to fully reinsure the firm's claims. In contrast, in a fixed-costs setting it may very well happen that refinancing is deemed too costly, in which case the manager's optimal behavior reverts to the Højgaard and Taksar setting. We characterize the threshold cost level above which no equity issuance takes place, given the firm's expected profitability, the volatility of its net returns and the investors' discount rate. Furthermore, we make precise how the boundary points x^{**} and x^* of the benchmark depend on the cost structure of the refinancing option. This is in contrast with He and Liang (2008). These authors study, in a proportional-costs setting, the situation where the firm's manager may vary the level of proportional reinsurance and may also decide to issue additional equity. They, however, take the boundaries x^{**} and x^* from Højgaard and Taksar (1998) as given and assume forced recapitalization should the firm be near bankruptcy. They then analyze under which conditions it is optimal to issue equity. This leads to situations where the firm may default strategically.

Natural extensions to Jeanblanc-Picqué and Shiryaev (1995) were considered by Déscamps et al. (2011) and by Lokka and Zervos (2008). These authors allow, respectively, for the possibility of equity issuance under fixed and proportional or only under proportional transaction costs. This introduces a means to ward off bankruptcy, as proportional reinsurance is ruled out.[2] In both cases conditions under which equity issuance is optimal are provided, but a closed-form solution to the manager problem is only provided in Lokka and Zervos (2008). The latter is not possible for the model studied in Déscamps et al. (2011) because interest accrues on accumulated reserves at a non-negative rate. This rate, however, is strictly smaller than the risk-less rate, which is a proxy for agency costs. The authors provide a qualitative description of the value function. They also describe the impact of financial frictions on firm governance, as well as how these governance issues affect the volatility of stock returns.

We have chosen a setting without a profitable saving technology because it allows for closed-form solutions to the value functions and, more interestingly, for (almost) fully analytical expressions for the free boundaries. As a consequence, we provide conditions on the reinvestment costs under which the partial-exposure region disappears, and we are also able to study the convergence properties of the value functions to the first-best case (when costs decrease) and to the solution in Højgaard and Taksar (1998) (when reinvestment becomes prohibitively costly). Our model extends Højgaard and Taksar (1998) by considering reinvestment possibilities and Déscamps et al. (2011) and Lokka and Zervos (2008) by allowing a variable exposure to the risky investment opportunity.

The remainder of the paper is organized as follows: we describe in Section 2 the Benchmark Model (Højgaard's and Taksar's) without reinvestment options, which we use as an initial building block. The case of proportional issuance costs is studied in Section 3, whereas fixed costs are introduced in Section 4. For the sake of completeness, a brief description of the solution to the Benchmark Model is provided in the Appendix, where all our mathematical proofs can also be found.

2. The benchmark

We work in a continuous-time, infinite-horizon setting and consider an insurance firm whose manager has the possibility to continuously adjust the proportion of claims that are reinsured. The manager's objective is to maximize the value of equity. The latter is defined as the expected, discounted dividend stream that the (risk-neutral) shareholders receive over the firm's lifespan.

In order to describe the dynamics of the cash flows generated by the exposure to risk, as well as the dividend payments, let us introduce the filtered probability space $(\Omega, \mathcal{A}, \mathcal{F}, \mathbb{P})$. The process $W = (W(t),\ t \geq 0)$ is a $\mathbb{P}$-Brownian motion that generates the filtration $\mathcal{F} = (\mathcal{F}_t,\ t \geq 0)$. Let μ, s_0 and σ be greater than zero, then the net income generated by full risk exposure is given by the stochastic differential equation

$$dS(t) = \mu dt + \sigma dW(t), \qquad S(0) = s_0.$$

Given that μ is greater than zero, the firm is profitable. It may, however, incur in both operating profits and losses, since σ is positive. The manager's decisions regarding risk exposure are represented by a predictable process $a = (a(t),\ t \geq 0)$, which we shall call an *exposure strategy*, that satisfies $a(t) \in [0, 1]$ for all $t \geq 0$. A *cumulative dividends process* is any non-decreasing, adapted càglàd process $L = (L(t),\ t \geq 0)$. Notice that the only restriction we impose on dividends is that they should be non-negative, which reflects the shareholders' limited liability. For a given choice of a and L, the dynamics of the firm's *cash reserves* are described by the stochastic differential equation

$$dR^{a,L}(t) = a(t)(\mu dt + \sigma dW(t)) - dL(t), \quad R^{a,L}(0) = x > 0.$$

A priori s_0 need not match x; thus an exceptional, lump-sum dividend being distributed at zero is not excluded. We have assumed, for simplicity, that no interest is earned on cash

[2] In Déscamps et al. (2011), the authors do not describe an insurance setting, but a corporate-finance one.