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Liquidity management of foreign exchange reserves in continuous time

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ABSTRACT

In order to cope with daily foreign currency exchange payments or trades and avoid liquidity crisis, central banks need to maintain the liquidity of foreign exchange reserves. In this paper, we develop a Foreign Exchange Reserves Liquidity Management (FERLM) model based on stochastic process by introducing a foreign exchange factor. We also generate a feasible target proportion of the liquidity reserve to total foreign exchange reserves, by seeking the balance between capital gains of holding foreign exchange reserves and losses of liquidity insufficiency.

1. Introduction

With the gradual deepening of economic globalization and the violent increasing of international trades, the impact of exchange rate flexibility upon the level of foreign exchange reserves is growing in recent years. Given the potential influence on economy and financial stability, the issue of efficient foreign exchange reserves management should be of considerable importance. Especially in the 21st century, big expansion takes place in the scales of foreign exchange reserves in most countries, therefore the liquidity problems of foreign exchange reserves are becoming much more vital. Consequently, central banks have to pay more attention to reasonable allocation of foreign exchange reserves.

Among different central banks, the management methods of foreign exchange reserves are different. For example, the foreign exchange rate policy of China is mandatory settlement of exchange. Foreign currency inflows are collected to the central bank, and become foreign exchange reserves. Moreover, because the exchange rate of China is not completely free floating, keeping the foreign exchange rate stabilization is not the main aim of the central bank. For this kind of counties as China, the foreign exchange reserves management method of central banks is approximately similar to the deposit management of commercial banks which can be divided into two categories.

One is to invest for obtain gains and to prevent from the foreign exchange devaluation. As far as the way of investment is concerned, many scholars gave their own answers (Joachim et al. (2006), Topaloglou et al. (2008), etc.).

The other is to maintain the liquidity to deal with daily payments and exchange transitions. Sufficient liquidity could reduce the risk of shortage liquidity, ensure the safety of financial system, and improve the economic development steadily. However, the excessive liquidity of foreign exchange reserves may also bring high holding costs.

The logic behind the liquidity management model of commercial banks (Ringbom et al. (2004), Xavier and Rochet (2008)) is to calculate a reasonable proportion of liquidity holdings based on cash inflows and outflows, however, until now this model is still constructed by stochastic variable. For a very short time interval, the stochastic variable model could well describe the cash inflows and outflows. Nevertheless, for a longer time interval, the cash flow and other factors are hugely influenced by time factor. This means that the stochastic variable model ignores process changes, and as a result will lead to lack of accuracy. Under this situation, a better choice would be to use a stochastic process.

In this paper, we build a Foreign Exchange Reserves Liquidity Management (FERLM) model of central banks, in which the foreign exchange rate is added as a new factor and a stochastic process is used on the basis of the liquidity management model of commercial banks. Under the consideration of liquidity crisis, the proposed model enables central banks to gain the maximum income.

In order to show the difference between central banks' FERLM model and commercial banks' deposit management model, in Section 2, we make a comparison of the two models through constructing a simple

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stochastic variable FERLM model. After assembling a stochastic process model based on gamma process (Dickson and Waters (1993), etc.), we explain the differences between the stochastic variable model and the stochastic process model.

In Section 3, by using exponential distributions to model the variables in the stochastic variable model and assuming that foreign exchange rate is constant, we obtain an analytical solution. Additionally, based on the obtained conclusion, we develop this stochastic variable model to a mixture model with respect to the foreign exchange rate.

In the stochastic process model, considering that foreign currency inflows and outflows are monotone un-decrease processes, we suppose they follow gamma processes. Because foreign currency flows are influenced by economic anticipation and foreign exchange rate anticipation of the domestic country, we add some economic factors (GDP and foreign exchange rate anticipations) to parameters of the gamma processes. Furthermore, we describe the foreign exchange rate by using a geometric jump diffusion process with the jump size following normal distribution, see Merton (1976).

In Section 4, we make simulations to examine these models proposed in Section 2, and show the differences.

Section 5 gives our main conclusions of this study.

2. Liquidity management models

The foreign currency funds, which are transferred in a country and exchanged for domestic currency, will be invested or speculated into various domestic markets. After a period, this money will be converted into foreign currency and flow out the country. During the whole process, the foreign exchange rate is a quite important factor that may cause more damages to the country's economy and foreign exchange reserves. In this section, under these circumstances, we try to build two FERLM models in a unit time interval $[t, t+1)$ on the foundation of the liquidity management model of commercial bank.

One is called stochastic variable model, in which there is only one-time inflow and outflow of foreign currency during the unit time interval $[t,\ t+1)$. The other is called stochastic process model, in which all factors are functions with respect to time t. Moreover, depending on the situation that central bank will not frequently adjust investing position of the foreign exchange reserves, we assume that there is only one-time adjustment of investing position and liquidity reserves during the unit time interval $[t,t+1)$, and the adjustment time is at t. Without loss of generality, suppose $t=0$.

2.1. Stochastic variable model

Let G_0 represent the total foreign exchange reserves in a central bank at time 0. X represents the payment of domestic currency, and e_0 is the foreign exchange rate, then Xe_0 is the payment of foreign currency during $[0,1)$. α is the liquidity proportion of the total foreign exchange reserves G_0. G represents the foreign inflow during $[0,1)$. r_L and r_p represent the return rate from the investment and the financing interest rate caused by liquidity shortage, respectively.

It is clear that the return of foreign exchange reserves of central bank is from investment, and the loss comes from liquidity shortage. Therefore our model is given as follows:

$$g(\alpha) = r_L G_0(1-\alpha) - r_p E[max\{0, Xe_0 - G - G_0\alpha\}], \tag{2.1}$$

where, G, X and e_0 are independent and G_0 is a constant. The first term on the right hand side of Eq. (2.1) represents the investment return, and the second term is the financing cost by liquidity shortage. If the foreign exchange rate factor e_0 is excluded from Eq. (2.1), the model becomes the liquidity management model of commercial banks.

It is known that the function $g(\alpha)$ is convex (Xavier and Rochet (2008)). Make $g(\alpha)$ be the largest with respect to α, which is represented as $max\{g(\alpha)\}$. Differentiate the formula (2.1) for α, and make it equal to zero, then the following result is obtained:

$$\frac{dg(\alpha)}{d\alpha} = -r_L G_0 + r_p G_0 Prob(Xe_0 - G > G_0\alpha) = 0.$$

Thus, we have

$$Prob(Xe_0 - G < G_0\alpha) = \frac{r_L}{r_p}, \tag{2.2}$$

where $r_L < r_p$.

2.2. Stochastic process model

Similar to the previous section, the total foreign exchange reserves are represented by G_0 at time 0. G, X, r_L, r_p and e_0, which are functions with respect to time t, $t\in[0,1)$, are represented as $G(t)$, $X(t)$, $r_L(t) = r_L t$, $r_p(t) = r_p t$ and $e(t)$ $(e(0)=e_0)$, respectively. Let $G(t)$ and $X(t)$ be monotone un-decrease stochastic processes, that is, $dG(t), dX(t) \geq 0$ and $G(0)=0$, $X(0)=0$. The total outflow of foreign currency is expressed as $\int_0^1 e(t)dX(t)$ and the total inflow is $\int_0^1 dG(t) = G(1)$ during the time interval $[0,1)$.

Let $Y(t)$ be net liquidity demand at time t, and it is represented as follows:

$$Y(t) = \int_0^t e(s)dX(s) - G(t) - G_0\alpha.$$

If $sup\{Y(t); t\in[0,1)\} \leq 0$, then it means that during the time interval $[0,1)$ the liquidity reserves holdings could satisfy the demands of foreign currency outflow and do not need any financing. Moreover, there would be not any liquidity crisis but rather liquidity surplus occurred.

If $sup\{Y(t) + G_0\alpha; t\in[0,1)\} \leq 0$, which means $\alpha = 0$, then the foreign currency inflow could be enough to satisfy the demand of foreign currency outflow during the time interval $[0,1)$. There would be clearly no need for financing and no liquidity shortage.

Obviously, $Y(0) = -G_0\alpha$, and $inf\{Y(t); t\in[0,1)\} \leq 0$. Otherwise if $inf\{Y(t); t\in[0,1)\} \geq 0$, then $\alpha = 0$, which shows that the foreign currency outflow exceeds the inflow, so liquidity shortage perhaps arises, and the central bank correspondingly needs to adjust the proportion α.

Fig. 1 shows a sample path of $Y(t)$ $(t\in[0,1))$. Because central banks would set aside liquid reserves $G_0\alpha$, external financing and correspondingly financing costs would not be generated until the funds of liquid reserves are completely used up and the liquidity shortage occurs. The financing cost of stochastic variable model could be regarded as the rectangle area surrounded from 0 to $Y(1)$ multiplying r_p in Fig. 1.

Then our stochastic process model based upon Eq. (2.1) during the time interval $[0,1)$ is given as follows:

$$g(\alpha) = (1-\alpha) r_L G_0 - r_p E\left[\int_0^1 max\{0, Y(t)\}dt\right],$$

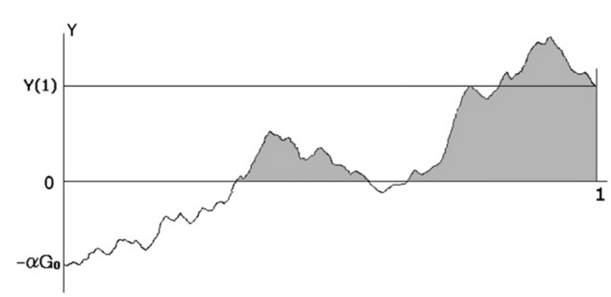

Fig. 1. Financing costs.