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Overlapping risk adjusted sets of priors and the existence of efficient allocations and equilibria with short-selling ☆

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Abstract

The theory of existence of equilibrium with short-selling is reconsidered under risk and ambiguity modelled by risk averse variational preferences. No-arbitrage conditions are given in terms of risk adjusted priors. A sufficient condition for existence of efficient allocations is the overlapping of the interiors of the risk adjusted sets of priors or the inexistence of mutually compatible trades, with non-negative expectation with respect to any risk adjusted prior. These conditions are necessary when agents are not risk neutral at extreme levels of wealths. It is shown that the more uncertainty averse or risk averse the agents, the more likely are efficient allocations and equilibria to exist.
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1. Introduction

The issue of the relationship between agents' beliefs and risk tolerances and the existence of efficient allocations and equilibria has first been considered, in the early seventies, by Grandmont [14], Green [15] and Hart [18] for markets with short-selling in the context of temporary equilibrium models and assets equilibrium models and reconsidered later by Hammond [16] and Page [25,26]. In these early models, investors were assumed to have a *single* homogeneous or heterogeneous probabilistic belief and be risk averse expected utility maximizers (EU). The Hammond, Hart, Page's (HHP) model has later been generalized to a model with consumption sets unbounded below. Sufficient conditions of no arbitrage were given for existence of an equilibrium (see e.g. Allouch et al. [1], Dana et al. [7], Nielsen [24], Page et al. [27], Page and Wooders [28] and Werner [33]). The no-arbitrage conditions were shown to be necessary for existence of equilibrium under adequate assumptions. More recently, risk sharing of an aggregate capital betwen different investors using convex measures of risk gave rise to problems of efficiency with short-selling (see e.g. Heath and Ku [19], Dana and Le Van [8], Barrieu and El Karoui [2], Filipovic and Svindland [12] and Jouini et al. [20]).

This paper reconsiders the equilibrium theory of assets with short-selling when there is risk and ambiguity. The variational preferences axiomatized by Maccheroni, Marinacci and Rustichini [23] (denoted MMR from now on) are used. Variational preferences nest many of the models developed to study ambiguity in the decision theoretic, financial and economic literature, in particular, the maxmin expected utility of Gilboa and Schmeidler [13], the penalty preference functionals of Hansen and Sargent [17] and the convex measures of risk introduced in mathematical finance (see Föllmer and Schied [11] for an overview). A risk averse variational preference is characterized by a convex cost (penalty) function defined on the probability simplex and a concave utility index that models risk-aversion. Up to a minus sign, convex measures of risk correspond to a risk neutral agent with a zero discount rate. Without loss of generality, attention may be restricted to the probabilities with finite cost that we call the *priors*. To simplify as much as possible the analysis, we assume complete markets and consider a standard Arrow–Debreu model of state contingent claims.

While in the HHP model, the useful trading directions have been characterized in the early seventies as trades fulfilling the *incomplete mean* condition by Bertsekas [3] and Hart [18], the no-arbitrage conditions have never been made explicit. The first contribution of the paper is the characterization, for MMR preferences, of the useful trading directions, the no-arbitrage prices, the concept of collective absence of arbitrage, in terms of *risk adjusted sets of priors*. The second contribution of the paper is to provide under the no half-line condition, necessary and sufficient conditions for existence of efficient allocations or equilibria: a necessary and sufficient condition is either that the intersection of the interiors of the risk adjusted sets of priors is non-empty or the inexistence of mutually compatible trades with non-negative expectations with respect to any risk adjusted prior. The condition that the intersection of the interiors of the risk adjusted set of priors is non-empty generalizes the conditions given in the early seventies for single beliefs. An equilibrium does not exist if agents disagree "very much". This happens if agents consider as relevant disjoint subsets of states of the world. For equilibrium to exist, agents must have sets of priors with overlapping supports, where by support, we mean a state of the world with a strictly positive probability for some prior. Unfortunately, even when this condition is fulfilled, there may not be an equilibrium if their sets of priors are too different. As written by Hart [18] when agents are very risk averse, strong disagreement on expectations may be compatible with the existence of an equilibrium.