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The rise of complex phenomena in Cournot duopoly games due to demand functions without inflection points

Sameh Askar *

Department of Statistics and Operations Researches, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia
Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

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ABSTRACT

In this paper, we propose Cournot duopoly games where quantity-setting firms use nonlinear demand functions that have no inflection points. Two different kinds of repeated games are introduced based on rationality process of firms and Puu's incomplete approach. First, a model of two rational firms that are in competition and produce homogenous commodities is introduced. The equilibrium points of this model are obtained and their dynamical characteristics such as stability, bifurcation and chaos are investigated. By using rationality process firms do not need to solve any optimization problem but they adjust their production based on estimation of the marginal profit. Using Puu's incomplete information approach a new model is introduced. As in the first model, the equilibrium points are obtained and their dynamical characteristics are investigated. By using Puu's approach firms only need to know their profits and the quantities produced in the past two times. We compare the properties of the two models under the two approaches. The paper extends and generalizes the results of other authors that consider similar processes.

1. Introduction

Economically, the notion of Nash equilibrium in firms games depends on the amount of knowledge competed firms know about the market. Each firm has to know information about the market (such as relevant prices in order to make optimal decisions) where it operates and knows what the other firms in the market decide to perform. In literature, many researchers have investigated that when market information is limited the economic agents' behavior may be divided into two important directions [1]. The first direction depends on some market experiments to extrapolate information using simple rules of thumb. In the second direction, interaction between past decisions and market mechanisms are conjecturally analyzed. Clower [2] has described the dilemma of the monopolist when facing an unknown demand function. Indeed, when a gradient rule is used the monopolist decides a quantity given the variation of the profits as it does not know the demand function. Baumol and Quandt [3] suggested a simple rule of thumb (the gradient rule) that a monopolist can use to estimate the demand [1]: if the profits are varied positively, the monopolist should change the price in the same direction from that of the preceding one and vice versa. Recently Naimzada and Ricchiuti [4] have shown that complex dynamical characteristics such as bifurcation and chaos, in a monopoly with rule of thumb, can be achieved with a cubic demand function that has no inflection point. Askar in [5] has investigated recently such complex characteristics using the gradient rule. He has used a generalized demand function of the one used in [4] and has observed that with this general form of demand different dynamic behaviors from stability to chaos through bifurcation have been emerged.

* Address: Department of Statistics and Operations Researches, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia. Tel.: +966 558757119.

E-mail addresses: saskar@ksu.edu.sa, s.e.a.askar@hotmail.co.uk

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In the second direction, monopolistic price setting was incorporated in a general equilibrium model for the first time in a brilliant paper by Negishi [6]. He introduced a concept called subjective demand curve. This curve has given the maximum quantity of a monopolized good that the monopolist thinks he can sell as a function of his price (price maker), given his market observations. He has argued that such subjective demand function is clearly much more realistic that the assumption on which the monopolist knows the exact demand function. Therefore, each time the monopolist should take a price decision based on re-estimating his subjective demand as function of what he observes. After that he chooses the prices of the commodities in the light of his subjective demand and his technological possibilities. Even though the subjective demand function is based on the observations of actual sales possibilities, the entire process of such price setting is somewhat unrealistic. It is based on completely fictitious observations besides it is not obvious where the subjective demand function comes from, with so few observations.

Bounded rationality and Puu's incomplete information are two different frameworks that have been recently used to study monopoly and duopoly markets. Bounded rational players (firms) update their production strategies based on discrete time periods and by using a local estimate of the marginal profit. With such local adjustment mechanism, the players are not requested to have a complete knowledge of the demand and the cost functions [7]. All they need to know is if the market will response to small production changes by an estimate of the marginal profit. This adjustment mechanism, which sometimes called myopic [7] has been extensively used by many authors, mainly with continuous time [8]. However, it is argued elsewhere [7] that a discrete time decision process is more realistic since in real economic systems, production decisions cannot be revised at every time instant. On the other hand, Puu's [9] has recently introduced the so-called Puu's incomplete information. It has main advantage that is it is realistic since a firm does not need to know the form of the profit function to get an estimate of the quantity produced in the next time step [10,11]. Instead all it needs is its profit and the quantities produced in the past two times. Recently, Ahmed et al. [12] have reported that systems based on Puu's techniques are numerically unstable when approaching to the equilibrium position. Moreover such systems have serious instabilities in the case of duopoly. The authors in [12] have modified those systems based on Puu's techniques with a change in the quantities produced by 10% per time step to avoid singularities in such systems.

The difference between the discrete approach represented in [1,9] and the continuous approach represented in [3] is due to different economic meaning of the left hand side of the two equations representing the adjustment mechanism in [1,9,3] respectively, that is the difference between dynamic evolution in continuous and in discrete time. In the discrete approach, the decisional mechanism proposed proceeds changing the volume of production without any graduation: if two subsequent production volumes belong to the same trajectory, all productions between the two are not experimented during the evolution of time. This means that production changes from one period to the next are proportional to the speed of adjustment, exogenously given, and to the marginal profit. On the other hand, in continuous time models, if two subsequent production volumes belong to the same trajectory, all productions between the two are experimented during the evolution of time, thus the speed of adjustment and the marginal profit determine the speed with which all the productions are implemented. It can be stated that the continuous time dynamic represents those decision-making processes characterized by very gradual changes, while the discrete time is associated with decision-making processes in which there are sudden and drastic changes in the volume of production. In the light of these considerations, it is also understandable that in the framework of one-dimension dynamic systems, in continuous time described in [3] we have always convergence to the optimal output. On the other hand, in discrete time with strong reactivity and/or radical changes in the volume of production, we may have a richer dynamical behavior of the system.

The goal of this paper is to set up a discrete time dynamic Cournot model in which duopoly firms use rule of thumb and Puu's incomplete information techniques to detect the equilibrium quantities to produce. In order to determine the effects of different variables of choice between firms, we set up a model characterized by a demand function that does not have an inflection point. We generalize, in this paper, the demand function developed by Naimzada and Ricchiuti [4]. Through simulations we have observed that with such cubic demand functions, there are different dynamic behaviors from stability to chaos through a sequence of period-doubling bifurcation.

The paper is devoted as follows: In Section 2, the model and the main results are illustrated and discussed in detail. Finally, some concluding results are presented.

2. Model and main results

Here, the inverse demand function presented by Naimzada and Ricchiuti [4] is generalized and used throughout the paper. The purpose of using this function is that the function has no inflection points. Moreover it has been investigated [4,5] that in the monopolist market it seems that Nash equilibrium goes from stability through double bifurcation to chaos when firms adopt cubic demand function. To start our work in the present paper, we presume that market includes only two firms produce the same commodities for sale in the market. Apparently, one should be aware that those two firms are in conflict (more precisely, in competition). Now, let us study the case of two firms in the market in which both firms use the following inverse demand functions.

$$P(Q_t) = a - bQ_t^3, \tag{1}$$