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On the control of some duopoly games

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ABSTRACT

The stabilization of the Cournot equilibrium point of a duopoly by feedback control is analyzed. We give a general result which is valid for any duopoly controlled by linear feedback and study in detail the stabilization of a duopoly when the feedback is not linear.

1. Introduction

A duopoly is a microeconomic model in which two firms evolve in a market. The firms produce goods that are prefect substitutes, and then, by assuming different demand functions we get different kinds of markets. For instance, if the demand function is linear and the marginal costs are constant, we receive that the firms planify their production following the rule

$$\begin{cases} q_1(t+1) = \dfrac{a-c_1}{2b} - \dfrac{1}{2}q_2(t), \\ q_2(t+1) = \dfrac{a-c_2}{2b} - \dfrac{1}{2}q_1(t), \end{cases} \quad (1)$$

where c_1 and c_2 are marginal costs, $q_1(t)$ and $q_2(t)$ are the outputs of any firm at time t, and $D = b - a(q_1 + q_2)$ is the linear demand function (see e.g [1] or [2]). In this case, the maps

$$f_1(q_2) = \frac{a-c_1}{2b} - \frac{1}{2}q_2,$$
$$f_2(q_1) = \frac{a-c_2}{2b} - \frac{1}{2}q_1,$$

are obtained by maximizing the profit of both firms and therefore are used for planifying the production. This is the reason why they are called reaction functions. Both firms maximize their profits at the fixed point, also called Cournot point, given by

$$\bar{q}_1 = \frac{a+c_2-2c_1}{3b},$$
$$\bar{q}_2 = \frac{a+c_1-2c_2}{3b}.$$

In this case, it is simple to see that Cournot point is stable and moreover, for any initial production of any firm, following (1) the productions converge to Cournot point.

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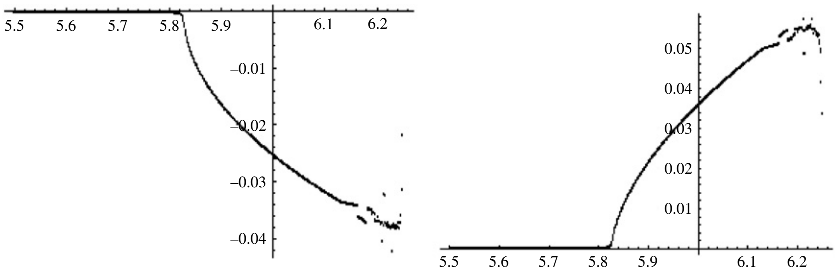

Fig. 1. Residual profit for firm 1 (left) and 2 (right). We see that firm 1 want to stabilize the Cournot equilibrium when $c_2 \geq 3 + 2\sqrt{2}$.

However, there are another possible assumption for demand functions and marginal costs which produces different duopoly markets and for which the above does not work: the Cournot point can be distabilized and therefore the final production need not converge to the Cournot point (see e.g. [3], [4] or [5]).

Hence, in [6] is proposed to control the Cournot point by using linear and nonlinear feedback. In particular, they obtain conditions for stabilizing the Cournot point of the classical Puu's duopoly (see [3]). In this paper, we propose to make an analysis of the stabilization, by using linear feedback, of Cournot equilibrium points of a general duopoly of the form

$$\begin{cases} q_1(t+1) = f_1(q_2(t)), \\ q_2(t+1) = f_2(q_1(t)), \end{cases} \tag{2}$$

where f_1 and f_2 are the reaction functions of the market. As a result, we find a gap in one of the main results from [6]. In addition, we propose to control the duopoly given by Norin and Puu in [4] by using nonlinear feedback.

A motivation for controlling duopoly games as presented before is the following. Consider the duopoly market given in [3], with constant marginal costs $c_1 = 1$ and $c_2 \in [5.5, 6.25]$. Let $(\overline{q}_1, \overline{q}_2)$ be the Cournot point, which will be stable if $c_2 \in [1, 3 + 2\sqrt{2}]$. If $\pi_i(q_1, q_2)$, $i = 1, 2$, denote the profit of firms when they produce q_1 and q_2, respectively, then Fig. 1 gives us an estimation of the residual average profit

$$\frac{1}{n}\sum_{t=0}^{n-1} \pi_i(q_1(t), q_2(t)) - \pi_i(\overline{q}_1, \overline{q}_2).$$

As we can see, when c_2 increases, the firm 1 loses because of the distabilization of the Cournot point. So, firm 1 is interested in stabilizing such equilibrium point, while we find the opposite for the firm 2.

The paper is organized as follows. Next section is devoted to study the stabilization of Cournot points by using linear feedback in the general case. Last section is devoted to analyze Norin and Puu duopoly by assuming that linear control is introduced in a nonlinear way.

2. Controlling a general duopoly: linear feedback

Assume the duopoly has the form

$$\begin{cases} q_1(t+1) = f_1(q_2(t)), \\ q_2(t+1) = f_2(q_1(t)), \end{cases}$$

where production of both firms in time t is $q_1(t)$ and $q_2(t)$, and f_1 and f_2 have sufficient derivative conditions. Let $(\overline{q}_1, \overline{q}_2)$ be a fixed Cournot point, that is,

$$(\overline{q}_1, \overline{q}_2) = (f_1(\overline{q}_2), f_2(\overline{q}_1)),$$

and let $a = f_1'(\overline{q}_2)$ and $b = f_2'(\overline{q}_1)$. Hence, the Jacobian matrix at $(\overline{q}_1, \overline{q}_2)$ is given by

$$\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix},$$

and therefore the fixed point is stable provided that $|a \cdot b| < 1$. Note that the characteristic polynomial of the Jacobian matrix is $t^2 - x$, where $x = a \cdot b$.

Usually, duopoly models are stabilized via adjustment processes which involve both firms (see e.g. [3] and the comments and references in regard with this question in [6]. Fig. 1 shows an example showing that, at least one firm is not interested in the system stabilization because it takes profit of Cournot equilibrium destabilization. However, the other firm, which has the lowest marginal cost and hence is the most efficient one, is interested in stabilizing the system. The feedback method that we will introduce below shows how this firm can stabilize the system in a "easy" way by increasing or decreasing its production in each unit of time.