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Original Article

Nonlinear dynamics in a Cournot duopoly with isoelastic demand

Luciano Fanti [a,1], Luca Gori [b,*], Mauro Sodini [a,2]

[a] Department of Economics and Management, University of Pisa, Via Cosimo Ridolfi, 10, I-56124 Pisa (PI), Italy
[b] Department of Law, University of Genoa, Via Balbi, 30/19, I-16126 Genoa (GE), Italy

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Abstract

This paper analyses the dynamics of a nonlinear Cournot duopoly with general isoelastic demand (quasi-linear preferences) and quantity-setting firms that have incomplete information about the market demand. Unlike existing papers, we propose a model where the price elasticity of demand is different from one. This causes interesting local and global dynamic events that cannot be observed in the case of unit-elastic demand and homogeneous players. In particular, the global behaviour of the map is studied through the critical curves technique, and numerical simulations show coexistence of attractors, coordination failures and complex structures of the basins of attraction.

JEL classification: C62; D43; L13

Keywords: Cournot; Isoelastic demand; Nonlinear dynamics

1. Introduction

Analysing nonlinear dynamics in oligopoly models dates back at least to Puu [21], later followed, amongst others, by Kopel [18], Puu [22,23] and Bischi et al. [6,7,9]. These papers have renewed interest in the study of expectations formation mechanisms differing from the rational expectations paradigm, as claimed by Agliari et al. [1, p. 527]: “When one takes into account the fact that nonlinear dynamical systems can produce dynamic paths that are not so regular and predictable, one of the major arguments against adaptive expectations does not seem so strong.” For instance, Bischi et al. [8] consider a Cournot duopoly with profit-maximising firms and incomplete information that estimate a linear demand function. In particular, they compare the behaviour of firms that use the Local Monopolistic Approximation adjustment process with the behaviour of firms that use an adjustment mechanism based on the Best Reply dynamics (where firms are assumed to have complete knowledge of the market demand with naïve expectations). They show, for the case of unit-elastic demand, that less information causes more stability to occur.

In addition, the idea that slight differences amongst producers’ characteristics (e.g., heterogeneities in the cost function or different behaviours of players depending on the information set about the profit function) may cause major

* Corresponding author. Tel.: +39 010 209 95 03; fax: +39 010 209 55 36.
E-mail addresses: fanti.luciano@gmail.com, lfanti@ec.unipi.it (L. Fanti), luca.gori@unige.it, dr.luca.gori@gmail.com (L. Gori), m.sodini@ec.unipi.it (M. Sodini).
[1] Tel.: +39 050 22 16 369; fax: +39 050 22 10 603.
[2] Tel.: +39 050 22 16 234; fax: +39 050 22 10 603.

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nonlinear local and global dynamic phenomena (such as lack of synchronisation and coexistence of attractors), has opened up the way for several interesting papers dealing with the behaviour of firms in a nonlinear oligopoly context (e.g., Tramontana et al. [26,27]).

In this class of models, the cases of both linear demand (quadratic utility) and unit-elastic demand (Cobb–Douglas utility) are analysed with different objectives. The former assumption (linear demand) is illustrated by Kopel [18], Bischi et al. [6,9] and Fanti et al. [16]. In particular, Kopel [18] shows that periodic and complex dynamics may take place in a model with cost functions that incorporate an inter-firm externality. Bischi et al. [9] concentrate on problems of synchronisation and intermittency that derive from a two-dimensional asymmetric map in a model with profit-maximising firms, while Fanti et al. [16] introduce managerial (relative profit) delegation and find that, despite the assumption of homogeneous players (symmetric map), a high degree of competition between managers may cause on–off intermittency, blow-out phenomena and multistability that are impossible under profit maximisation with homogeneous players. The latter assumption (unit-elastic demand) demand is illustrated by Puu [21,23], Bischi et al. [6], Agliari et al. [2], Tramontana [25] and Gao et al. [17]. All these papers hypothesise unit-elastic demand on the part of consumers. In particular, Bischi et al. [6] show that symmetry-breaking bifurcations can occur only when some heterogeneities exist in the main parameters of the model, while Tramontana [25] explores a model characterised by one firm with complete knowledge of the profit function and myopic expectations about its rival's output decisions, and the other firm with no knowledge of the profit function that makes use of local estimation of its marginal profits to adjust production in the future period. The main finding of the paper is that the interior fixed point of a two-dimensional map may undergo either a flip bifurcation or Neimark-Sacker bifurcation.

The aim of the present paper is to characterise the local and global dynamic properties of a nonlinear Cournot duopoly with general isoelastic demand and price elasticity different from one (quasi-linear preferences) by assuming that firms use local estimates of marginal profits to adapt production decisions period by period. Unlike Bischi et al. [6], although players are homogeneous and thus the dynamics of the model is described by a two-dimensional symmetric map, we show that interesting global phenomena can be observed when the price elasticity of demand is different from one.

The rest of the paper is organised as follows. Section 2 describes the static model. Section 3 introduces a dynamic mechanism of production. Section 4 studies the properties of the basins of attraction. Section 5 introduces and describes critical curves. Section 6 studies local stability, global dynamic properties and synchronisation phenomena, and Section 7 concludes. Appendix provides microeconomic foundations for the general isoelastic demand function.

2. The model

We consider a Cournot duopoly with two types of agents: firms and consumers. A homogeneous good is produced and traded on a market with general isoelastic demand given by $p = Q^{-1/\eta}$ (which derives from quasi-linear preferences),[3] where p is the consumers' marginal willingness to pay for product $Q = q_1 + q_2$, $q_1 \geq 0$ and $q_2 \geq 0$ are outputs produced by firms 1 and 2, respectively, and $\eta > 0$ is a parameter that captures the degree of demand elasticity. The case $\eta = 1$ (log-linear preferences) represents the unit-elastic demand model analysed, amongst others, by Bischi et al. [6], Puu [21] and Tramontana [25] in a Cournot duopoly under different hypotheses with regard to the information set of both players. In particular, Bischi et al. [6] assume that both players have limited information regarding profits (no knowledge of the market) and every firm behaves adaptively following a local estimate of their own profits (bounded rationality), Puu [21] assumes that that players have myopic expectations, both having complete knowledge of the profit function, while Tramontana [25] considers the heterogeneous case of myopic expectations and bounded rationality.

The production function of firm $i = \{1, 2\}$ has constant marginal returns to labour, that is $q_i = L_i$, where L_i represents the labour force employed by firm i (Correa-López and Naylor [14]). Firm i's cost function is assumed to be $C_i(q_i) = c_i L_i = c_i q_i$, where c_1 and c_2, with $c_1 \geq c_2 > 0$, represent the constant average and marginal costs of producing an additional unit of output for firm 1 and firm 2, respectively.

Firm i maximises profits (Π_i) that are given by:

$$\Pi_i = (p - c_i)q_i = (Q^{-1/\eta} - c_i)q_i = [(q_i + q_{-i})^{-1/\eta} - c_i]q_i. \tag{1}$$

[3] See Appendix for microeconomic foundations of the general isoelastic demand function used in this paper.