Available online at www.sciencedirect.com

ScienceDirect

Procedia Economics and Finance 19 (2015) 146 – 153

Procedia Economics and Finance

www.elsevier.com/locate/procedia

The Economies of Balkan and Eastern Europe Countries in the changed world, EBEEC 2014, Nis, Serbia

On the dynamics of a duopoly game with differentiated goods

Georges Sarafopoulos *

Department of Economics, Democritus University of Thrace, Komotini, 69100 Greece.

Abstract

The present study extends Fanti, Gori, 2012 (L. Fanti, L. Gori, 2012. The dynamics of a differentiated duopoly with quantity competition, Economic Modelling, 29, 421-427). In this study we investigate the dynamics of a nonlinear discrete-time duopoly game with differentiated goods, linear demand and quadratic cost functions. The game is modeled with a system of two difference equations. Existence and stability of equilibria of this system are studied. We show that the model gives more complex chaotic and unpredictable trajectories as a consequence of change in the parameter of horizontal product differentiation and a higher (lower) degree of product differentiation (weaker or fiercer competition) destabilize (stabilize) the economy. The chaotic features are justified numerically via computing Lyapunov numbers and sensitive dependence on initial conditions. Also, we show that in the case of quadratic costs there are stable trajectories and a higher (lower) degree of product differentiation does not tend to destabilize the economy.

Peer-review will be under responsibility of Department of Accountancy and Finance, Eastern Macedonia and Thrace Institute of Technology, Kavala, Greece.

Keywords: Product differentiation; Discrete Dynamical System; Equilibrium; Stability; Chaotic Behavior.

1. Introduction

An Oligopoly is a market structure between monopoly and perfect competition, where there are only a few number of firms in the market. The dynamic of an oligopoly game is more complex because firms must consider not only the behaviors of the consumers, but also the reactions of the competitors i.e. they form expectations concerning how

* Corresponding author. Tel.: +302531039819; fax: +302531039830.
E-mail address: gsarafop@ierd.duth.gr

Peer-review will be under responsibility of Department of Accountancy and Finance, Eastern Macedonia and Thrace Institute of Technology, Kavala, Greece.
doi:10.1016/S2212-5671(15)00016-7

their rivals will act. Cournot, in 1838 has introduced the first formal theory of oligopoly. He treated the case with naive expectations, so that in every step each player (firm) assumes the last values that were taken by the competitors without estimation of their future reactions. In particular, we consider differentiated products and focus on the dynamic role played by the degree of horizontal product differentiation. While Cournot 1838 considered a duopoly with a single homogeneous product, more recently the economic literature offered duopoly models with (horizontal) differentiated products (see, e.g., Dixit, 1979; Singh and Vives, 1984) which allow goods and services to be substitutes or complements, in models with a standard linear demand structure.

Expectations play an important role in modelling economic phenomena. A producer can choose his expectations rules of many available techniques to adjust his production outputs. In this paper we study the dynamics of a duopoly model where each firm behaves with different expectations strategies. This kind of beliefs is common in real world problems such as economic, biology and social sciences problems. We consider a duopoly model where each player forms a different strategy in order to compute his expected output. We take firm 1 to represent a boundedly rational player while firm 2 has naive expectations. Each player adjusts his outputs towards the profit maximizing amount as target by using his expectations rule. Some authors considered duopolies with homogeneous expectations and found a variety of complex dynamics in their games, such as appearance of strange attractors (Agiza, 1999, Agiza et al., 2002, Agliari et al., 2005, 2006, Bischi, Kopel, 2001, Kopel, 1996, Puu,1998). Also models with heterogeneous agents were studied (Agiza, Elsadany , 2003, 2004, Agiza et al., 2002, Den Haan , 20013, Fanti, Gori, 2012, Tramontana , 2010, Zhang , 2007).

The main result of the present analysis is that the proposition of Fanti Gori, 2012, is not true if the cost function is quadratic. In this case there are trajectories of the system in which an increase in product differentiation may destabilize the unique Cournot–Nash equilibrium, but we show that in this case there are stable trajectories and a higher (lower) degree of product differentiation does not tend to destabilize (stabilize) the economy. Moreover, from a mathematical point of view, we show that the destabilization of the fixed point can occur through a flip bifurcation and also that a cascade of flip bifurcations may lead to periodic cycles and deterministic chaos.

The paper is organized as follows: In Section 2, the dynamics of a differentiated duopoly game with heterogeneous expectations, linear demand and quadratic cost functions is analyzed. The existence, local stability and bifurcation of the equilibrium points are also analyzed. In Section 3 numerical simulations are used to show complex dynamic via computing Lyapunov numbers, and sensitive dependence on initial conditions.

2. The game

2.1. Construction of the game

In oligopoly game players can choose simple expectation rules such as naïve or complicated as adaptive expectations and bounded rationality. The players can use the same strategy (homogeneous expectations) or can use different strategy (heterogeneous expectations). In this study we consider heterogeneous players such that each player think with different strategy to maximize his output. Two different players expectations are proposed; bounded rational player and naive player. We consider a simple Cournot-type duopoly market where firms (players) produce differentiated goods and offer them at discrete-time periods t = 0, 1, 2, … on a common market. At each period t, every firm must form an expectation of the rival's output in the next time period in order to determine the corresponding profit-maximizing quantities for period t+1. If x (resp. y) is the quantity of product of variety 1 (resp. 2), the inverse demand functions (as a function of quantities) are given by the following equations (Fanti, Gori, 2012)

$$p_1 = a - x - dy,\ p_2 = a - y - dx, \tag{1}$$

We suppose that the cost functions are quadratic:

$$C_1(x) = cx^2,\ C_2(y) = cy^2 \tag{2}$$

where a, b , c >0, -1 < d < 1.
d represents the degree of horizontal product differentiation. More in detail, if d=0, then goods and services of