



# Nonlinear dynamics in a heterogeneous duopoly game with adjusting players and diseconomies of scale

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## ABSTRACT

A repeated, discrete time, heterogeneous Cournot duopoly game with bounded rational and adaptive players adjusting the quantities of production is subject of investigation. Linear inverse demand function and quadratic cost functions reflecting decreasing returns to scale are assumed. The game is modeled with a system of two difference equations. Evolution of outputs over time is obtained by iteration of a two dimensional nonlinear map. Existing equilibria and their stability are analyzed. In face of diseconomies of scale, bounded rational and adaptive duopolists are shown to experience a decrease in the latitude of their output adjustment decisions with respect to the market stability compared to constant returns to scale and *ceteris paribus*. Chaotic dynamics is confirmed to depend mainly on the adjustment behavior of the bounded rational player, who if overshoots leaves the adaptive player with limited opportunities to stabilize the market again, hence industries facing diseconomies of scale are found to be less stable than those with constant marginal costs. Complexity of the dynamical system is examined by means of numerical simulations, where the paper extends the results of other authors who considered analogous games assuming linear cost functions. Intermittent transition to chaos and attractor merging crisis are shown among others.

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## 1. Introduction

Oligopoly is a market structure between monopoly and perfect competition. It is characterized by a domination of several firms, which completely control trade. These firms manufacture homogeneous products. They have to consider both the market demand, i.e. the behavior of consumers, as well as the strategies of their competitors, i.e. they form expectations concerning how their rivals will act. The most widely used and simultaneously the first formal model of oligopoly market was proposed by Cournot in 1838. The French mathematician investigated a duopoly case, where two firms were naive players. He assumed that each company adjusts its quantity of production to that of its rival and there is no retaliation at all, so that in every step duopolist perceives the latest move made by the competitor to remain his last. However, it is highly unlikely that all players share naive expectations. Therefore, different approaches to firm behavior were proposed. Players were not only perceived to be naive, but also bounded rational and adaptive. Works on Cournot model showed that it has an ample dynamical behavior under different expectations.

Some authors considered duopolies with homogeneous expectations and found a variety of complex dynamics in their games, such as appearance of strange attractors with fractal dimensions [1,2,5,6,25,33,35]. They demonstrated that dynamics of Cournot oligopoly games may never converge to equilibrium and in the long term bounded periodic or chaotic behavior

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may be observed. Also models with heterogeneous agents were studied [3,4,19,26,37,38]. Agiza and Elsadany [4] applied the method of Onazaki et al. [30], who examined stability, chaos and multiple attractors of a heterogeneous two dimensional cobweb model, to investigate dynamics of a duopoly game with bounded rational and naive players facing constant marginal costs. Zhang et al. [38] evidenced chaotic behavior in a modification of the game by Agiza and Elsadany [4] involving quadratic instead of linear cost functions and thus increasing marginal costs. Other authors considered bivariate forms of cost functions to reflect interfirm externalities demonstrating chaos in Cournot duopoly models as well [10,25]. In turn Tramontana [37] amended the game by Zhang et al. [38] removing nonlinearity from the cost function, at the same time introducing a microfounded one into the demand function, thereby allowing for higher number of different routes leading to complex dynamics.

In this paper nonlinear, in particular quadratic, cost functions are introduced into the study of a duopoly game considered in Agiza and Elsadany [3], where costs of bounded rational and adaptive players were modelled in a linear manner, leading to constant marginal costs assumption. Quadratic cost functions reflect decreasing returns to scale or diseconomies of scale and are frequently met in applications, for instance in the modeling of renewable resources exploitation, such as fisheries [11]. Introduction thereof lets theoretical implications of the model be generalized over industries facing continuously increasing marginal costs of production. Within the proposed framework, duopolists concerned about stability of the market are found to experience a considerable loss in flexibility of adjusting their respective outputs, compared to operating under constant marginal costs. Complex dynamics is confirmed to depend mainly on the adjustment behavior of the bounded rational player. In case the bounded rational player overshoots significantly, opportunities for the adaptive player to stabilize the market are limited when marginal costs increase. Transition to chaos is demonstrated to occur through a cascade of flip bifurcations, an intermittency and a crisis.

The paper is organized in the following way. In Section 2 different expectations rules are presented and the best reply functions are discussed. Section 3 is an analysis of the nonlinear duopoly game with bounded rational and adaptive players, modelled with a two dimensional noninvertible map. Section 4 includes numerical simulations, applied to demonstrate complexity of the dynamical system. Finally, Section 5 concludes the obtained results.

## 2. Expectations and best replies

Three expectations rules are commonly observed in the analyses of diverse oligopoly games. Simple naive on one hand versus sophisticated adaptive and bounded rational beliefs on the other. These tenets can be found in both homogeneous and heterogeneous agents paradigms. A simple, heterogeneous Cournot duopoly setup is considered here. Given system comprises of two firms,  $X$  and  $Y$ . Company  $X$  and company  $Y$  produce homogeneous, perfectly substitutive goods, in quantities  $x$  and  $y$  respectively. Goods are offered at discrete time periods  $t = 0, 1, 2, \dots, \infty$ , on a shared market. At each period  $t$  companies  $X$  and  $Y$  form expectations of each other's output in period  $t + 1$ . Firms' outputs for the latter period are decided by solving the following optimization problem

$$\begin{cases} x_{t+1} = \arg \max \pi^x(x_t, y_{t+1}^e) \\ y_{t+1} = \arg \max \pi^y(x_{t+1}^e, y_t) \end{cases} \quad (1)$$

where  $\pi^x(\cdot)$  and  $\pi^y(\cdot)$  denote expected profit functions of firms  $X$  and  $Y$  accordingly, and  $x_{t+1}^e, y_{t+1}^e$  represent their respective beliefs about competitor's future production decision. Unique solutions to (1), if exist, are denoted by

$$\begin{cases} x_{t+1} = f(y_{t+1}^e) \\ y_{t+1} = g(x_{t+1}^e) \end{cases} \quad (2)$$

where  $f$  and  $g$  are reaction functions, also referred to as best replies to  $y_{t+1}^e$  and  $x_{t+1}^e$  respectively. Naive expectations rule assumes that each firm expects the rival to offer the same quantity for sale in the current period, as he did in the previous one. Hence,  $x_{t+1}^e = x_t$  for company  $X$  and  $y_{t+1}^e = y_t$  for  $Y$  [17]. After implementing Cournot theory, (2) becomes

$$\begin{cases} x_{t+1} = f(y_t) \\ y_{t+1} = g(x_t) \end{cases} \quad (3)$$

which describes a homogeneous duopoly game with naive agents, where Cournot  $f$  and  $g$  reaction functions are linear.

Conversely to naive players some firms may rationally utilize information from the market. In a static Cournot oligopoly model, fully informed rational players are assumed to reach the Nash equilibrium immediately, in one shot. Unfortunately such an assumption is invalid in the real world because information obtained from the market is usually incomplete. Therefore, notion of bounded rationality is employed in dynamic Cournot oligopoly models [12]. Bounded or not fully rational players do not possess complete knowledge of the market. If firm  $X$  is assumed to be a bounded rational player, though ignorant about market demand, it updates its production strategy by an adjustment mechanism based on a local estimate of the marginal profit  $\Phi_t = \frac{\partial \pi^x(x_t, y_t)}{\partial x_t}$  [7], denoted also as gradient dynamics [9] or myopic adjustment mechanism [20]. Then, agent  $X$  behaves as a local profit maximizer. At each time period  $t$  firm  $X$  decides to increase or decrease the output for period  $t + 1$  if it experiences positive or negative marginal profit on the basis of information held at time  $t$ , according to the following dynamic adjustment mechanism

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