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Equilibrium stability of a nonlinear heterogeneous duopoly game with extrapolative foresight

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Abstract

We make a further attempt to investigate equilibrium stability of a nonlinear Cournot duopoly game with adaptive adjustment toward best reply by assuming heterogeneous firms where one firm only uses naive expectations whereas the other employs a simple forecast technology to form sophisticated expectations. More precisely, based on the knowledge of actual production of the competitor and its actual rate of change, the clever firm is able to evaluate its opponent's output in the near future by virtue of straightforward extrapolative foresight. We finally arrive at a conclusion that this seemingly rational mechanism takes a positive effect on convergence to equilibrium behavior. Inconsistent with common intuition, we demonstrate that stronger foresight ability is not always better to stabilize the equilibrium. Particularly, perfect foresight dose not give rise to the best stabilizing factor.

Keywords: Cournot game; Adaptive adjustment; Short-term extrapolative foresight; Equilibrium stability; Local bifurcations

1. Introduction

Complex dynamic behavior involving chaos is typically undesirable in traditional economic systems where stationary states are mainly focused on [8]. Many economically plausible mechanisms are introduced in order to improve the stability of economic dynamics. For example, Shamma and Arslan [10] propose such a learning mechanism that each player adjusts its belief by considering the opponent's strategic setting in the near future under two famous classes of evolutionary game dynamics, the fictitious dynamic as well as the gradient dynamic. They demonstrate that convergence to Nash equilibria can always be achieved in an ideal case of exact derivative measurements. Quandt [9] adopts a short-term forecast method to enhance the stability of a continuous adaptive adjustment dynamic in a Bertrand game. Kamalinejad et al. [7] show in Cournot competition the stability of a discrete adjustment dynamic can be always guaranteed by taking into account linear regression and recursive weighted least-squares expectations. Besides, Huck et al. [6] reconsider the best reply dynamic by incorporating inertia to virtually slow the adjustment process and to ultimately ensure the stability of Cournot equilibrium.

In this paper, we investigate equilibrium stability of a nonlinear Cournot duopoly game with heterogeneous firms, who update their outputs adaptively toward best reply [1,2,5]. In particular, we assume one firm is endowed with naive expectations so that it believes the rival's output to be invariable. The other firm, on the other hand, is assumed

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to be capable of realizing the adversary's short-term output by actual adjustment of the competitor and its actual rate of change. We analytically and numerically study the influence of proposed short-term extrapolative foresight on convergence to equilibrium behavior. We indicate that our modified model is "more stable" for a larger range of parameter set. In other words, this seemingly rational decision rule is proved to be a stabilizing factor on the stability of Cournot equilibria. We further indicate that there exists an optimal foresight rule, where the stability space is extended to the greatest extent. It can be therefore concluded that more powerful foresight is not always better for convergence to equilibrium behavior.

The paper is organized as follows. As a benchmark, a basic adjustment dynamic and its stability analysis are given in Section 2. By introducing the short-term extrapolative foresight, our model is constructed and stability analysis is provided in Section 3. In Section 4, numerical simulations follow, while Section 5 concludes our paper.

2. The basic model

Let x_t and y_t denote the outputs of the first firm and the second in discrete time period t, respectively. The general adaptive adjustment model is given by

$$\begin{cases} x_{t+1} = (1-\omega_1)x_t + \omega_1 R_1(y^e_{t+1}) \\ y_{t+1} = (1-\omega_2)y_t + \omega_2 R_2(x^e_{t+1}) \end{cases}, \qquad (1)$$

where $\omega_i \in (0, 1]$ measures firm i's adjustment speed, R_i represents firm i's reaction function, y^e_{t+1} and x^e_{t+1} denote the first firm's expectations and the second's expectations toward their competitors' outputs, respectively. The case of naive expectations is widely studied where both firms expect their rivals' outputs to be unchangeable, i.e., $x^e_{t+1} = x_t$, $y^e_{t+1} = y_t$.

The reaction function of Kopel's adjustment dynamic [3] takes the following form

$$\begin{cases} R_1(y_t) = \mu y_t(1-y_t) \\ R_2(x_t) = \mu x_t(1-x_t) \end{cases},$$

where both firms are assumed to have naive expectations and $\mu > 0$ characterizes the magnitude of the positive externality that one firm has on the other's payoff. Hence, the adaptive adjustment model above reduces to

$$\begin{cases} x_{t+1} = (1-\omega_1)x_t + \omega_1 \mu y_t(1-y_t) \\ y_{t+1} = (1-\omega_2)y_t + \omega_2 \mu x_t(1-x_t) \end{cases}. \qquad (2)$$

Similar with the work by Bischi and Kopel [3], in this paper we always assume $\mu \in [1, 4]$ and only focus on the following nontrivial fixed points in the strategy space $[0, 1] \times [0, 1]$,

$$E_s = \left(1 - \frac{1}{\mu}, 1 - \frac{1}{\mu}\right) \quad \text{for} \quad \mu \in (1, 4],$$

$$E_1 = (x^*, y^*) \quad \text{and} \quad E_2 = (y^*, x^*) \quad \text{for} \quad \mu \in (3, 4],$$

where $x^* = ((\mu + 1 + \sqrt{(\mu+1)(\mu-3)})/2\mu)$ and $y^* = ((\mu + 1 - \sqrt{(\mu+1)(\mu-3)})/2\mu)$. Obviously, these fixed points coincide with Cournot equilibria.

Denote

$$\Omega = \{(\mu, \omega_1, \omega_2) | 1 \le \mu \le 4,\ 0 < \omega_1 \le 1,\ 0 < \omega_2 \le 1\}.$$

As a benchmark, the key results by Bischi and Kopel [3] are summarized in the following lemma.

Lemma 1. *Under dynamic (2), E_s is asymptotically stable for the parameter space*

$$\Omega^s_0 = \{(\mu, \omega_1, \omega_2) \in \Omega | 1 < \mu < 3\}.$$