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Nonlinear dynamics and global analysis of a heterogeneous Cournot duopoly with a local monopolistic approach versus a gradient rule with endogenous reactivity

Fausto Cavalli [a,*], Ahmad Naimzada [a], Fabio Tramontana [b]

[a] Department of Economics, Management and Statistics, University of Milano-Bicocca, Milan, Italy
[b] Department of Economics and Management, University of Pavia, Italy

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ABSTRACT

We study a heterogeneous duopolistic Cournotian game, in which the firms, producing a homogeneous good, have reduced rationality and respectively adopt a "Local Monopolistic Approximation" (LMA) and a gradient-based approach with endogenous reactivity, in an economy characterized by isoelastic demand function and linear total costs. We give conditions on reactivity and marginal costs under which the solution converges to the Cournot–Nash equilibrium. Moreover, we compare the stability regions of the proposed oligopoly to a similar one, in which the LMA firm is replaced by a best response firm, which is more rational than the LMA firm. We show that, depending on costs ratio, the equilibrium can lose its stability in two different ways, through both a flip and a Neimark–Sacker bifurcation. We show that the nonlinear, noninvertible map describing the model can give rise to several coexisting stable attractors (*multistability*). We analytically investigate the shape of the basins of attractions, in particular proving the existence of regions known in the literature as *lobes*.

1. Introduction

Starting from the seminal book of Cournot in 1838 [1], the oligopolistic market structures have been deeply studied and analyzed as a significant subject of economic dynamics and game theory. In an oligopolistic framework, the market is supplied by only a few firms, two in the particular case of a duopoly, and each firm chooses its strategy taking into account at the same time both its own actions and those of their competitors.

Assuming that each firm in endowed with an high degree of rationality and with cognitive and computational skills that allow them to exactly know the demand curve of the produced good and their technology (represented by the cost function), an oligopoly can be studied in a static game setting by means of the notion of Nash equilibrium. Having each firm perfect foresight of the next period productions, the players are able to solve a one period optimization problem.

Weakening the degree of rationality of the firms, a dynamic adjustment arises and new scenarios appear, as even a simple duopoly can give rise to complex dynamic phenomena, as shown in the works of Rand [2] and Poston and Stewart [3] in 1978. The dynamic behavior of oligopolistic models have been intensively investigated: among all the contributions, we can mention the key work of Puu in 1991 [4], which proposed a duopoly based on unimodal reaction functions derived

* Corresponding author.
E-mail addresses: fausto.cavalli@unimib.it (F. Cavalli), ahmad.naimzada@unimib.it (A. Naimzada), fabio.tramontana@unipv.it (F. Tramontana).

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solving an optimization problem for profit functions. Studying the case of constant marginal costs and, starting from Cobb-Douglas type preferences for the consumer, of isoelastic demand function, Puu showed that each competitor outputs can evolve through a period doubling sequence of flip bifurcations which leads finally to chaos.

The work of Puu started a wide literature about duopolistic and oligopolistic market and lead off different research strands. One of these, deeply investigated in the last years, concerns the analysis of decisional mechanism, dissimilar to the best response rule, based on different (usually lower) degrees of informational and computational abilities. Among these mechanisms, we can mention the gradient-like mechanism and ‘‘Local Monopolistic Approximation’’ (LMA).

The LMA is an approach which was proposed for the first time by Silvestre [5] with the name of ‘‘Strong Monopolistic Equilibrium’’, and then reconsidered by Tuinstra [6], Bischi et al. [7], Naimzada and Sbragia [8] and applied in a monopolistic setting by Naimzada and Ricchiuti [9]. In the LMA, the oligopolists do not know the market demand function, and conjecturing it is linear, they proceed estimating such a linear function through the knowledge of the local true slope of the demand curve and the knowledge of the current market state in terms of quantities and price. We remark that Silvestre [5] already introduced such concept.

In the gradient-like mechanism, the players, which are not requested to have a complete knowledge of the demand and cost function, do not choose their strategy solving any optimization problem, but they use a local estimate of the marginal profit. The output level is updated by following the direction of an increase of the corresponding profit function, which is indeed regulated by its gradient. The reactivity of this adjustment is governed by a parameter, which can be set in different ways. In particular, it can be constant with respect to the produced quantity or endogenously dependent from it, to enforce its dependence on the firm size. The constant reactivity gradient mechanism is discussed in the book of Varian [10] and by Corchon and Mas-Colell [11], while duopolies involving a firm following it were studied by Angelini et al. [12] and Cavalli et al. [13]. Conversely, the endogenous reaction was studied by Bischi and Naimzada [14], Bischi et al. [15,16], Agiza et al. [17], Tramontana [18] and more recently by Askar [19,20]. In the present work, we consider endogenous reaction.

This paper belongs to the literature about heterogeneous oligopolies, in the sense that the firms adopt different decisional mechanisms. The heterogeneous framework was studied for example in the articles by Leonard and Nishimura [21], Den-Haan [22], Agiza and Elsadany [23,24], Angelini et al. [12], Tramontana [18], Dubiel-Teleszynski [25] and Anufriev et al. [26]. These works concern the coupling of a best response decisional mechanism with the gradient like decisional mechanism. In our work, we want to consider a duopoly made up by two firms which adopt the gradient-like and the Local Monopolistic Approximations. This work belongs to the research strand in which we are investigating several aspects of heterogeneous duopolies. In particular, we focus on the effect of different degrees of rationality in Cavalli and Naimzada [27], where we compared decisional mechanisms based on best response adjustments involving different degrees of rationality. Conversely, in Cavalli et al. [13] we study the effect of exogenous reactivity.

We remark that the economic structure in which we study our model is similar to those proposed by Angelini et al. [12] and Tramontana [18] with respect to the isoelastic demand function and to the constant marginal costs. In particular, the duopoly we consider differs from that studied in [18] only for the second firm decisional mechanism, since in [18] the second firm chooses its output using the classical best response with static expectations, so the two duopolies are directly comparable.

The main results of this work concern the study of the local stability of the equilibrium; the investigation of the possible routes of destabilization for the equilibrium point; the study of global properties of the two dimensional map that models the oligopoly.

Concerning the first result, after obtaining the constraints on the costs ratio to have a stable equilibrium, we compare the resulting stability region of the Nash equilibrium with that of the duopoly studied in [18]. We prove that, despite of the reduced rationality of one of the firms, the duopoly studied in this work can present a larger stability region. In particular, the stability region is larger for the duopoly studied in the present work when the costs are sufficiently different, while it is larger for the gradient versus best response duopoly when the costs are more comparable.

Concerning the second result, we show that the equilibrium point can be destabilized by means of both a flip and a Neimark–Sacker bifurcation, depending on the costs ratio. In particular, it is shown that, when the cost ratio is sufficiently favorable to the firm which adopts the gradient-like decisional mechanism, the scenario presents a Neimark–Sacker bifurcation, while the flip bifurcation occurs when the costs structure is favorable to the firm using linear approximation. This result improves the knowledges about dynamics involving gradient rule, and further confirms recent studies showing that both Neimark–Sacker and flip bifurcations appear to be the possible destabilization, as shown in [15,16,12,18,13], especially when a gradient-rule dynamic is present.

Concerning the third result, we show that dealing with a nonlinear, and in particular noninvertible map, the only local stability analysis does not permit to fully understand the dynamics. Several stable attractors may coexist (*multistability*) giving rise to a situation of *path dependency*. The shape of the basins of attractions can be quite complicated, especially because we deal with a two-dimensional map characterized by a denominator that can vanish along a line, and one component of the map takes the form 0/0 at one point. In these cases the basins of attraction can be characterized by particular shapes known in the literature as *lobes*.

The paper is organized as follows. In Section 2, we introduce the model and the nonlinear system describing the dynamics of the productions of the firms. In Section 3, we determine the conditions under which the Nash equilibrium is locally stable. In Section 4, we investigate the global behavior of the system. In Section 5 we conclude and we propose some future developments. In Appendix A we collect the proof of the propositions.