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Dynamic properties of a Cournot–Bertrand duopoly game with differentiated products

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ABSTRACT

In this paper we consider a Cournot–Bertrand duopoly model with linear demand and cost functions and with product differentiation. We propose a dynamic framework for the study of the stability properties of this kind of mixed oligopoly game, a rather neglected topic in the existing literature despite its relevance. In particular, in this paper we highlight the role of best response dynamics and of an adaptive adjustment mechanism for the stability of the equilibrium.

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1. Introduction

A huge number of papers deal with Cournot or Bertrand competition in oligopolistic markets. A considerably lower number of works are devoted to Cournot–Bertrand competition, characterized by the fact that the market can be subdivided into two groups of firms, the first of which optimally adjusts prices and the other one optimally adjusts their output. Nevertheless, works like Häckner (2000), Zanchettin (2006), Arya et al. (2008) and Tremblay et al. (2009) argue that in some cases Cournot–Bertrand competition may be optimal. Moreover, empirical evidence of this kind of competition is abundant (see, for instance, Tremblay et al., 2010). To the best of our knowledge Bylka and Komar (1976) and Singh and Vives (1984) are the first authors to analyze duopolies where one firm competes on quantities and the other on prices. This kind of competition requires a certain degree of differentiation between the products offered by the firms, in order to avoid that the whole market is served by the one that applies a lower price.

Recently Tremblay and Tremblay (2011) analyzed the role of product differentiation for the static properties of the Nash equilibrium of a Cournot–Bertrand duopoly.

The goal of this paper is to set up a benchmark framework that permits analysis of the dynamic properties of the Cournot–Bertrand competition. In order to easily determine the effects of the different variables of choice between the firms, we set up a model characterized by standard assumptions like the linearity of demand and cost functions. Moreover the assumption that one firm sets prices and the other one quantity is the only source of heterogeneity. Homogeneity is preserved both in technology and decisional mechanisms adopted by the competitors. This permits us to attribute to the special kind of competition the cause of the results we obtain. In particular, we study what happens when best response dynamics is introduced and when the firms are made more general by adding an adaptive adjustment mechanism. Our results are that under best response dynamics a lower level of differentiation is required for instability with respect to the Tremblay and Tremblay case. This critical degree of differentiation decreases the more the firms are reluctant to change their strategic choice from one period to the next one, i.e. they are characterized by more inertia, or an anchoring attitude.

The paper is organized as follows. In Section 2 we introduce the model in its static version. In Section 3 we study the local and global properties of the equilibria in a dynamic setting and introduce both instantaneous and adaptive adjustments. Some conclusions are presented in Section 4.

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2. The model

There are two firms in a market and firm i produces good x_i. Firms make their strategic choices simultaneously. Firm 1 competes in output q_1 as in a Cournot duopoly, while firm 2 fixes its price p_2 like in the Bertrand case. We assume profit maximizing firms that operate in an environment with complete information. The inverse demand functions are:

$$p_1 = a - q_1 - dq_2 \tag{1}$$

and

$$p_2 = a - q_2 - dq_1, \tag{2}$$

where $a > 0$ and $0 \le d \le 1$. The parameter d denotes the degree of product differentiation or product substitution. If $d = 1$, the two inverse demand functions become identical, which is the case of homogeneous goods. If $d = 0$, the two goods are independent and the duopoly market changes into two monopolistic markets. By assuming negative values of the parameter we would introduce complementarity between them. We prefer to limit our analysis to the case of substitutability. We can write the demand system in the two strategic variables, q_1 and p_2:

$$p_1 = a(1-d) - \left(1-d^2\right)q_1 + dp_2.q_2 = a - p_2 - dq_1 \tag{3}$$

The profit function for firm i is $\pi_i = (p_i - c)q_i$, for $i = 1, 2$, where c is the common, fixed marginal cost. Even if the assumption about common marginal costs can sound peculiar, we remark that the aim of this paper consists in finding the consequences of the heterogeneity in the variables of choice, avoiding the confusion that could be created by adding some other sources of heterogeneity. Solving the first-order conditions for profit maximization of π_i yields the following linear best reply functions:

$$r_1(p_2) : q_1 = \frac{a(1-d)-c}{2(1-d^2)} + \frac{d}{2(1-d^2)}p_2 . r_2(q_1) : p_2 = \frac{a+c}{2} - \frac{d}{2}q_1 \tag{4}$$

The best reply functions intersect in the Nash equilibrium (NE) point given by:

$$(q_1^*; p_2^*) = \left(\frac{(a-c)(2-d)}{4-3d^2}; \frac{a\left(2-d-d^2\right)+c\left(2+d-2d^2\right)}{4-3d^2}\right), \tag{5}$$

whose properties have been deeply analyzed in Tremblay and Tremblay (2011).

The aim of our work consists in studying the dynamic properties of the NE, in particular the conditions under which convergence is guaranteed.

Considering time, we suppose that the two firms take action in discrete time through their best replay functions with respect to the expected strategy of the other firm in the same period time. We suppose also that expectations are static. This hypothesis is preserved in the whole paper. Nevertheless, by introducing the adaptive adjustment, we weaken this hypothesis. In this case we suppose that the change of the strategic variables is proportional to the adjustment speeds, $0 < \alpha \le 1$ and to the distances between the current strategy and the firm''s best reply function. This creates some inertia or anchoring effect for the firm that is in some sense reluctant to change its current strategy. These cases are analyzed in the next section.

3. Global stability of the NE

3.1. Best reply with instantaneous adjustment

Usually firms do not have complete information about the choices of the competitor. In this case, in order to choose strategic variables, firms must form an expectation concerning the other firm's choice. Cournot proposed that, without any other information, firms may conjecture that the rival will make the same decision in the next period as taken in the current one. These kinds of expectations are known as static (or naïve) expectations. In our case, firm 1 supposes that firm 2 will maintain in $t+1$ the same price fixed in the current period t, and similarly firm 2 chooses the price considering the production of the firm 1 in $t+1$ equal to the same already produced in t. This means that the best responses with instantaneous adjustment are given by:

$$\begin{cases} q_1' = r_1(p_2) = \dfrac{a}{2(1+d)} - \dfrac{c}{2(1-d^2)} + \dfrac{d}{2(1-d^2)}p_2 \\ p_2' = r_2(q_1) = \dfrac{a+c}{2} - \dfrac{d}{2}q_1 \end{cases} \tag{6}$$

where (') denotes the unit-time advancement operator.

The only fixed point of the dynamical system (Huang, 2001) is the NE (Häckner, 2000), but the important question concerns the stability of this point.

Proposition 1. The Nash equilibrium with best reply and instantaneous adjustment is globally asymptotically stable provided that $d < \frac{2}{\sqrt{5}}$.

Proof. The Jacobian matrix of the system (Huang, 2001) is:

$$J : \begin{bmatrix} 0 & \dfrac{d}{2(1-d^2)} \\ -\dfrac{d}{2} & 0 \end{bmatrix} \tag{7}$$

with a trace equal to 0 and a positive determinant equal to:

$$Det = \frac{d^2}{4(1-d^2)} \tag{8}$$

This implies that the eigenvalues are purely imaginary and conjugate with modulus:

$$|\lambda| = \sqrt{\frac{d^2}{4(1-d^2)}} \tag{9}$$

which is inside the unit circle provided that:

$$\sqrt{\frac{d^2}{4(1-d^2)}} < 1 \Longleftrightarrow d < d_S = \frac{2}{\sqrt{5}}. \tag{10}$$

□

Proposition 1 states that, if the degree of substitution between the products is not excessively high, then the equilibrium is globally asymptotically stable.[1]

[1] Remember that we are not considering the case of complementarity between goods, that is the case $d < 0$

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