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Dynamics of a delayed duopoly game with bounded rationality

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ABSTRACT

A bounded rationality duopoly game with delay is formulated. Its dynamical evolution is analyzed. The existence of an economic equilibrium of the game is derived. The local stability analysis has been carried out. The analysis showed that firms using delayed bounded rationality have a higher chance of reaching a Nash equilibrium point. Numerical simulations were used to show bifurcation diagrams and phase portraits.

1. Introduction

An oligopoly is the case where the market is controlled by a small number of firms. Even the duopoly situation, an oligopoly of two producers, can be more complex than one might imagine since the duopolists have to take into account their actions and reactions when a decision is made. Oligopoly theory is one of the oldest branches of mathematical economics dating back to 1838 when its basic model was proposed by Cournot [1,2]. In the repeated oligopoly game all players maximize their profits. Recently, the dynamics of the duopoly game has been studied in [3–13]. Bischi and Naimzada [5] gave the general formula of the oligopoly model with a form of bounded rationality. They discussed the global and local stability of the duopoly game with a particular form of bounded rationality. They showed that the dynamics of the game can lead to complex behavior such as cycles and chaos. Ahmed et al. [13] used the Jury condition to discuss the stability of a modification of Puu's model with bounded rationality. Agiza et al. examined the dynamical behavior of Bowley's model with bonded rationality [12]. Agiza et al. [11] have been studied the complex dynamics of a bounded rationality duopoly game with a nonlinear demand function. The modification of the duopoly game depends on the strategy that the firms use, such as homogeneous and heterogeneous; and the expectations of the output the firms have to maximize, such as bounded rationality, naive expectation and adaptive expectation, see [10,9]. They developed duopoly game with heterogeneous players. The development of complex oligopoly dynamics theory has been reviewed in [14]. Other studies on the dynamics of oligopoly models with more firms and other modifications have been studied [15–17,6,18]. Also in the past decade, there has been a great deal of interest in chaos control of duopoly games because its complexity see [19,20] and its references.

The present work aims to formulate a bounded rationality duopoly game with delay and studying its dynamical behaviors. In additions it is aimed to check if the delay case used (that is, considering markets with memory) is a more realistic assumption than the non-delay case and increases the stability of the system.

This paper is organized as follows. In Section 2, the delayed duopoly game with bounded rationality is briefly described. In Section 3, we analyze the dynamics for a simple case of a delayed duopoly game with bounded rationality. Explicit parametric conditions of the existence, local stability of equilibrium points will be given. In Section 4, we present the numerical simulations, to verify our results which taken place by the theoretical analysis. Finally, some remarks are confined in Section 5.

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2. Delayed duopoly game with bounded rationality method

We consider a Cournot duopoly game where q_i denotes the quantity supplied by firm $i, i = 1, 2$. In addition let $P(q_i + q_j)$, $i \neq j$, denote to a twice differentiable and nonincreasing inverse demand function and let $C_i(q_i)$ denote the twice differentiable increasing cost function. For firm i the profit resulting from the above Cournot game is given by

$$\Pi_i(t) = P(q_i(t) + q_j^e(t+1))q_i(t) - C_i(q_i(t)). \tag{1}$$

The profit maximizing behavior of player i, taking the quantity supplied to the opponent j, $i \neq j$, as given, results in the well-known reaction function for firm i

$$q_i(t+1) = r_i(q_j^e(t+1)) = \arg\max_{q_i}[P(q_i(t) + q_j^e(t+1))q_i(t) - C_i(q_i(t))]. \tag{2}$$

Cournot assumed that the expected quantity in the next time step $q_j^e(t+1)$ is given by

$$q_j^e(t+1) = q_j(t). \tag{3}$$

Then the Cournot duopoly game defined as a discrete dynamical system has the form

$$q_i(t+1) = r_i(q_j(t)), \quad i, j = 1, 2, \quad i \neq j. \tag{4}$$

However, as pointed out by Bischi and Naimzada [5] there is an unrealistic assumption in this approach. It is implicitly assumed that the duopolist knows the market's demand function. A more realistic approach is to assume a bounded rationality i.e. each firm (say ith one) modifies its production according to its marginal profit $\frac{\partial \Pi_i(q_i(t), q_j^e(t+1))}{\partial q_i}$, $i = 1, 2, i \neq j$. Hence the dynamical system of a duopoly game with a bounded rationality is

$$\begin{cases} q_i(t+1) = q_i(t) + \alpha_i(q_i)\dfrac{\partial \Pi_i(q_i(t), q_j^e(t+1))}{\partial q_i} \; i = 1, 2, \quad i \neq j \end{cases} \tag{5}$$

where $\alpha_i(q_i)$ is the adjustment of the ith firm $i = 1, 2$. They assumed that $q_j^e(t+1) = q_j(t)$ in the term of the bounded rationality and also assumed $\alpha_i(q_i) = \alpha_i q_i$. Then the Bischi-Naimzada bounded rationality duopoly game has the form

$$\begin{cases} q_i(t+1) = q_i(t) + \alpha_i q_i \dfrac{\partial \Pi_i(q_i(t), q_j(t))}{\partial q_i} \; i = 1, 2, \quad i \neq j \, . \end{cases} \tag{6}$$

This means that, if the marginal profit is positive/negative he increases /decreases its production q_i in the next output period.

Also they assumed that the expected product of a firm $q^e(t+1)$ is equal to its previous quantity $q(t)$ in the bounded rationality term. However it may make more sense to use previous productions i.e. $q(t-1), q(t-2), \ldots, q(t-T)$ with different weights. This point of view has been studied in [13,21–23] in a different context. Ahmed et al. and Agiza et al. have been examined this point in the monopoly case only. In [13,21], they assumed that delay was put in the full term of bounded rationality for all players in the game. In this paper, we think that a greater reality for this game is put the delay in the term of the bounded rationality for all players except the ith player. Here both realistic ideas of bounded rationality and delay are combined. It will be shown that delay increases the stability domain. The dynamical system will be

$$q_i(t+1) = q_i(t) + \alpha_i q_i \frac{\partial \Pi_i(q_i, q^D)}{\partial q_i}, \quad i = 1, 2 \tag{7}$$

where $q^D = q_j^e(t+1) = \sum_{l=0}^{T} q_j(t-l)\omega_l$, $\omega_l \geq 0$, $\sum_{l=0}^{T} \omega_l = 1$. The factors ω_l, $l = 0, 1, 2, \ldots, T$ are the weights given to previous productions.

From Eq. (7), it is clear that the delay was put in the bounded rationality term for all players except the player i. This argument is the basic difference between our paper and the other papers [13,21,24].

3. Dynamics of the simple delayed duopoly game with bonded rationality

For simplicity set $T = 1$, and consider the duopoly case and the profit of i th firm is given by

$$\Pi_i = q_i(a - b(q_1 + q_2)) - c_i q_i, \quad i = 1, 2.$$

Under the above assumption, the delayed duopoly game with bounded rationality (7) is given by

$$\begin{cases} q_1(t+1) = q_1(t) + \alpha_1 q_1\{a - c_1 - 2bq_1(t) - b[\omega_2 q_2(t) + (1-\omega_2)q_2(t-1)]\} \\ q_2(t+1) = q_2(t) + \alpha_2 q_2\{a - c_2 - 2bq_2(t) - b[\omega_1 q_1(t) + (1-\omega_1)q_1(t-1)]\} \end{cases} . \tag{8}$$

To study the stability of dynamical system (8), rewrite it as a fourth dimensional system in the form

$$\begin{aligned} p_1(t+1) &= q_1(t) \\ p_2(t+1) &= q_2(t) \\ q_1(t+1) &= q_1(t) + \alpha_1 q_1(t)(a - c_1 - 2bq_1(t) - b[\omega_2 q_2(t) + (1-\omega_2)p_2(t)]) \\ q_2(t+1) &= q_2(t) + \alpha_2 q_2(a - c_2 - 2bq_2(t) - b[\omega_1 q_1(t) + (1-\omega_1)p_1(t)]). \end{aligned} \tag{9}$$