



ELSEVIER

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

SCIENCE @ DIRECT®

Physica A 320 (2003) 512–524

PHYSICA A

[www.elsevier.com/locate/physa](http://www.elsevier.com/locate/physa)

# Nonlinear dynamics in the Cournot duopoly game with heterogeneous players

H.N. Agiza\*, A.A. Elsadany

*Department of Mathematics, Faculty of Science, Mansoura University, P.O. Box 64,  
Mansoura 35516, Egypt*

Received 31 May 2002

---

## Abstract

We analyze a nonlinear discrete-time Cournot duopoly game, where players have heterogeneous expectations. Two types of players are considered: boundedly rational and naive expectations. In this study we show that the dynamics of the duopoly game with players whose beliefs are heterogeneous, may become complicated. The model gives more complex chaotic and unpredictable trajectories as a consequence of increasing the speed of adjustment of boundedly rational player. The equilibrium points and local stability of the duopoly game are investigated. As some parameters of the model are varied, the stability of the Nash equilibrium point is lost and the complex (periodic or chaotic) behavior occurs. Numerical simulations are presented to show that players with heterogeneous beliefs make the duopoly game behave chaotically. Also, we get the fractal dimension of the chaotic attractor of our map which is equivalent to the dimension of Henon map.

© 2002 Elsevier Science B.V. All rights reserved.

*PACS:* 87.23.Ge; 05.45.Gg; 05.45.Pq

*Keywords:* Discrete dynamical systems; Cournot duopoly games; Complex dynamics; Heterogeneous expectations

---

## 1. Introduction

An oligopoly is a market system which is controlled by a few number of firms producing homogeneous products. The dynamic of oligopoly game is more complex because oligopolist must consider not only the behaviors of the consumers, but also

---

\* Corresponding author.

*E-mail addresses:* [agizah@mans.edu.eg](mailto:agizah@mans.edu.eg) (H.N. Agiza), [aelsadany@mans.edu.eg](mailto:aelsadany@mans.edu.eg) (A.A. Elsadany).

the reactions of the other competitors. Cournot, in 1838 [1], introduced the first formal theory of oligopoly, who treated the case with naive expectations, so that in every step each player assumes the last values taken by the competitors without estimation of their future reactions.

Recently, several works have shown that the Cournot model may lead to complex behaviors such as cyclic and chaotic, see, for example Refs. [2–7]. Among the first to do this was Puu [3,4] who found a variety of complex dynamics arising in the Cournot duopoly case including the appearance of attractors with fractal dimension. Other studies on the dynamics of oligopoly models with more firms and other modifications include Ahmed and Agiza [8], Agiza [5] and Agiza et al. [9] such efforts have been extended by Bischi and Kopel [10] in a duopoly game with adaptive expectations. The development of complex oligopoly dynamics theory have been reviewed in Ref. [11].

Expectations play a key role in modelling economics phenomena. A producer can choose his expectations rules of many available techniques to adjust his production outputs. May be in the market of duopoly model each firm behaves with different expectations strategies, so we are going to apply this kind of expectations in our model which is common in reality.

In this paper we consider a duopoly model which is introduced in Ref. [7] but each player form a different strategy in order to compute his expected output. We take firm 1 represent a boundedly rational player while firm 2 has naive expectations. Each player adjusts his outputs towards the profit maximizing amount as target and use his expectations rule. Recently, examples of oligopoly games with homogeneous expectations are studied by Puu [4], Kopel [6], Agiza [12], Agiza et al. [13,14]. It was shown that the dynamics of Cournot oligopoly game may never settle to a steady state, and in the long run they exhibit bounded dynamic which may be periodic or chaotic. Economic model with heterogeneous players is introduced see [15,16]. Also, the dynamics of heterogeneous two-dimensional cobweb model have been studied by Onozaki et al., see Ref. [17].

The main aim of this work is to investigate the dynamic behaviors of a heterogeneous model representing two firms using heterogeneous expectations rules. This mechanism was applied in cobweb model [17] and gave us a guide to apply it in our study.

The paper is organized as follows. In Section 2 we describe the evolution of dynamical systems of players with heterogeneous expectations rules. In Section 3, the dynamics of a duopoly game with boundedly rational player and naive player is modelled by a two-dimensional map. The existence and local stability of the equilibrium points of the nonlinear map are analyzed. Complex dynamics of behavior occur under some changes of control parameters of the model which are shown by numerical experiments. Fractal dimension of the strange attractor of the map is measured numerically.

## 2. Heterogeneous expectations

In oligopoly game players choose simple expectations such as naive or complex as rational expectations. The players can use the same strategy (homogeneous expectations)

متن کامل مقاله

دریافت فوری ←

**ISI**Articles

مرجع مقالات تخصصی ایران

- ✓ امکان دانلود نسخه تمام متن مقالات انگلیسی
- ✓ امکان دانلود نسخه ترجمه شده مقالات
- ✓ پذیرش سفارش ترجمه تخصصی
- ✓ امکان جستجو در آرشیو جامعی از صدها موضوع و هزاران مقاله
- ✓ امکان دانلود رایگان ۲ صفحه اول هر مقاله
- ✓ امکان پرداخت اینترنتی با کلیه کارت های عضو شتاب
- ✓ دانلود فوری مقاله پس از پرداخت آنلاین
- ✓ پشتیبانی کامل خرید با بهره مندی از سیستم هوشمند رهگیری سفارشات