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Price vs quantity in a duopoly supergame with Nash punishments

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Abstract

We examine the endogenous choice between price and quantity behaviour in a duopoly supergame with product differentiation. We find that (i) if cartel profits are evenly split between firms, then only symmetric equilibria s obtained; (i) if instead the additional profits available through collusion are split according to the Nash bargaining solution, there are parameter regions where all subgame perfect equilibria are asymmetric, with firms colluding in price–quantity supergames.

Keywords: Repeated games; Collusion; Product differentiation

1. Introduction

A large body of literature has examined the relative stability of collusion in Bertrand and Cournot markets, parametrized over the degree of product differentiation.[1] Relatively few, instead, allow for the possibility that firms are asymmetric in terms of their market variables, considering thus price–quantity supergames (Lambertini, 1997; Albæk and Lambertini, 2004). In Rothschild (1995) and Lambertini and Schultz (2001, 2003), the possibility that firms optimally choose whether to be price- or quantity-setters in each period is considered. Notwithstanding the fact that firms are allowed to select different market variables, asymmetric cartels emerge from none of these contributions at the subgame perfect equilibria. Does this entail that antitrust agencies must not worry about such types of collusive behaviour? The underlying symmetry between firms, in terms of technology and product differentiation, which is a consistent feature of the models belonging to this stream of literature seems to provide an intuitive explanation for this result.

Moreover, neither Lambertini (1997) nor Albæk and Lambertini (2004) investigate which market variable yields the highest profit to the implicit cartel members for a given discount factor. Instead, they identify the lowest discount factor compatible with a subgame perfect equilibrium where firms stabilize collusion at the monopoly profits, being committed to set quantities or prices in all phases of the infinitely repeated duopoly game. Then, under the assumption

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[1] The number of contributions in this field is very large. See Deneckere (1983, 1984), Majerus (1988), Ross (1992), Rothschild (1992), Albæk and Lambertini (1998) and Lambertini and Sasaki (1999, 2002), *inter alia*.

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that firms are able to collude along the frontier of industry profits irrespective of the market variable(s) being set, a meta-game is introduced, whereby firms choose market variables once and for all, the payoffs being the smallest discount factors needed for sustaining the monopoly outcome in the repeated game, given the market variable(s). In this meta-game, firms are supposed to prefer small discount factors. These papers show that a prisoner's dilemma may arise in the meta-game where firms choose the market variable(s), i.e. that the latter may be inefficient.

The present paper nests in the existing literature on the stability of collusion and its relation to product differentiation. We partly rely on the analysis carried out in Lambertini (1997) and Albæk and Lambertini (2004), with a relevant departure from their line of research. That is, we still suppose that the choice of any given market variable is a long-running commitment to be taken at the outset in the meta-game,[2] but we assume that the relevant payoffs in the meta-game are given by discounted profit flows. We shall examine two different setups: one where cartel profits are split evenly between firms, and the other where, in asymmetric (i.e. price–quantity) supergames, the additional profits attainable through collusion are split according to the Nash bargaining solution. These two alternatives have largely different consequences in terms of subgame perfect equilibrium outcomes. In the game with equal split, only symmetric equilibria arise, and firms collude along the subgame perfect equilibrium path if and only if both of them are either price- or quantity-setters. The outcome is significantly different if collusive profits are split following a Nash bargaining solution in the asymmetric cases. If so, there exist parameter ranges (where product differentiation as well as firms' discount factors are sufficiently low) in which firms are indeed able to stabilize price–quantity cartels at the pure-strategy subgame perfect equilibria. That is, asymmetric market behaviour, if combined with Nash bargaining over cartel profits, helps firms collude in such a way that, when *a priori* symmetric firms are unable to sustain either Bertrand or Cournot cartels, they may nonetheless activate price–quantity cartels along the frontier of industry profits. In addition to the purely theoretical result *per se*, this argument also entails that asymmetry between firms helps stabilize collusion. This can be contrasted with a relatively small literature, where it is maintained that unequal firm sizes hinders the stability of tacit collusion (see, e.g., Compte et al. (2002)).

The remainder of the paper is structured as follows. The demand system is laid out in Section 2. Section 3 describes the meta-game. The critical discount factors for each possible supergame are listed in Section 4. The analysis of subgame perfect equilibria is in Section 5. Section 6 contains a few concluding remarks.

2. Setup

Two firms, labelled i and j, supply the market with a single product each. Firm i's inverse demand function is

$$p_i = 1 - q_i - \gamma q_j, \tag{1}$$

where $\gamma \in (0, 1]$ denotes the degree of substitutability between the two products. If $\gamma = 0$, firms are independent monopolists, therefore we shall exclude this case in the remainder of the analysis.

The direct demand function faced by firm i is:

$$q_i = \frac{1}{1+\gamma} - \frac{1}{1-\gamma^2} p_i + \frac{\gamma}{1-\gamma^2} p_j. \tag{2}$$

When instead firm i acts as a quantity-setter while firm j is a price-setter, their respective demand functions are:

$$p_i = 1 - q_i + \gamma(p_j + \gamma q_i - 1); \tag{3}$$

$$q_j = 1 - p_j - \gamma q_i. \tag{4}$$

For the sake of simplicity, we assume that firms operate at constant returns to scale and, without further loss of generality, we normalize the marginal cost to zero. Accordingly, each firm's profit function corresponds to revenue $\pi_i = p_i q_i$.

3. The meta-game

The concept of an extended game is due to Hamilton and Slutsky (1990, HS henceforth). They consider the extension, out of real time, of the basic duopoly game taking place in real time, in order to endogenize firms' choices

[2] An alternative strand of literature analyses supergames with capacity commitments, where the distinction between price and output strategies is less sharp, if not completely immaterial (see Brock and Scheinkman (1985), Benoit and Krishna (1987) and Davidson and Deneckere (1990)).