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Unexpected spin-off from quantum gravity

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Abstract

We propose a novel way of investigating the universal properties of spin systems by coupling them to an ensemble of causal dynamically triangulated lattices, instead of studying them on a fixed regular or random lattice. Somewhat surprisingly, graph-counting methods to extract high- or low-temperature series expansions can be adapted to this case. For the two-dimensional Ising model, we present evidence that this ameliorates the singularity structure of thermodynamic functions in the complex plane, and improves the convergence of the power series.

Keywords: Spin systems; Random geometries; Series expansion

1. Introduction

Quantum gravity is not usually regarded a subject of particular relevance to physics outside the exotic realm of Planck scale phenomena. Nevertheless, progress in science can sometimes come from an unexpected direction. In this letter, we will give a concrete example of how a specific way of constructing a theory of quantum gravity may lead to a new method for understanding the critical behaviour of certain spin and matter systems.

Apart from a handful of well-known exceptions, most thermodynamic properties of statistical mechanical models are not known to us in exact, closed form. Consequently, we must rely on a variety of approximation and numerical methods to study their behaviour. Lattice structures appearing in such models can either reflect the actual microscopic composition of a particular magnetic material, say, or play the role of a convenient discrete regulator for a continuous system whose symmetry properties have little to do with those of the lattice approximation.

Our focus will be on instances of the latter, where one is only interested in universal properties of the lattice models, which pertain to the system in a suitable scaling limit and are largely independent of discretization details, including those of the geometry of the underlying lattice. This is a point of view also encountered in the nonperturbative lattice formulations of both quantum gauge theories and gravity in high-energy physics.

In such a "utilitarian" view of lattice systems one is naturally led to ask how one should set up the lattice discretization to extract the desired continuum information in the quickest and most reliable way. Part of this

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quest is a systematic study of the influence of the lattice geometry on results, in an effort to separate as cleanly as possible 'universal behaviour' from 'lattice artefacts'. An example are the investigations of two-dimensional Ising models [1] on different regular lattices and their thermodynamic functions in the complex-temperature plane [2], which give clues on how to improve the convergence behaviour of approximation methods applied to high- and low-temperature expansions.

Going beyond regular lattices, and staying within the same philosophy, the use of *random lattices* has been advocated [3], in the hope that the absence of discrete lattice symmetries and therefore of distinguished lattice directions may accelerate the restoration of continuous rotational and translational invariances, and thus the approach to the continuum limit. However, we are not aware of any applications in lattice field theory where this would have led to a practical or conceptual breakthrough.

Taking yet a further step toward randomizing the spaces underlying the statistical models, one may consider an *additional averaging over random lattices*. This is inspired by quantum gravity, where the "path integral" (a nonperturbative quantum superposition of all spacetime geometries, which is central to the quantum dynamics) can be defined via a statistical, weighted sum over triangulated random geometries. The original approach of (*Euclidean*) *dynamical triangulations* (EDT—See Refs. [4,5] for reviews) turns out to be unsuitable for our purposes, because the contributing triangulated lattices are highly curved, with an effectively fractal structure, for any dimension $d \geqslant 2$. Their geometry is so radically different from the usual flat lattices that it alters the universal behaviour of matter systems defined on them, as has been well documented for two-dimensional spin systems (see Ref. [6] and references therein).

In fact, these geometries are so "wild" that they are not even suited for modelling the quantum behaviour of four-dimensional gravity. However, a promising new avenue has opened up recently with the advent of *causal dynamical triangulations* (*CDT*), which were exactly invented to fix the extreme geometric degeneracies of the previous, Euclidean approach (see Ref. [7] for a review). One still works with an ensemble of lattices with large local curvature fluctuations, but one where a partial order has been imposed in one of the lattice directions ("time"). In "pure" gravity (i.e., without matter coupling) this is sufficient to produce geometries whose effective (or Hausdorff) large-scale dimension d_H—in the sense of ensemble averages—equals the dimension d of their microscopic triangular building blocks, for $d = 2$ [8], $d = 3$ [9] and $d = 4$ [10], which was *not* the case for the corresponding Euclidean models, and is an indication that the geometries are much better behaved.

2. Are causal dynamical triangulations the "better lattices"?

We will in what follows concentrate on the model of causal dynamical triangulations in dimension $d = 2$, where the partition function of pure quantum gravity has been computed exactly [8]. There is no known exact solution in the presence of Ising spins, but CDT coupled to one [11] and eight [12,13] Ising models has been studied using Monte Carlo simulations.[1] A rather surprising outcome[2] of this analysis was that the behaviour of the matter is very robust: to high accuracy, the critical matter exponents coincide in both cases with those of the Onsager solution, despite large fluctuations of the underlying lattice geometry, and—in the case of the eight Ising copies—a shift in the geometry's Hausdorff dimension d_H from 2 to 3, due to the presence of the matter. This provides strong evidence (though not a proof) that the universal properties of any spin or matter system coupled to two-dimensional causal dynamical triangulations will be identical to those on a fixed, flat two-dimensional lattice. Since in terms of the degree of geometric disorder our model presumably lies in between the highly disordered EDT models and the more mildly disordered random Voronoi–Delaunay lattices, this would also lend credence to findings that the three-state Potts model on the latter exhibits a behaviour unchanged from the flat-lattice case [15].

Assuming that the universal flat-space properties of the matter systems do remain unaffected when coupled to causal dynamical triangulations, we want to advocate the CDT ensemble of triangulated geometries as a "background lattice" for studying their continuum behaviour. In view of the fact that these geometries are

[1] Regarding the lattice fluctuations as a type of disorder, the CDT-plus-matter models are examples of models with so-called *annealed* disorder, i.e., there is a genuine backreaction of the matter on the geometry.

[2] Especially for those closely familiar with the analogous Euclidean EDT results [14].