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Stackelberg solution in a vendor–buyer supply chain model with permissible delay in payments

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ABSTRACT

In practice, vendors (or sellers) often offer their buyers a fixed credit period to settle the account. The benefits of trade credit are not only to attract new buyers but also to avoid lasting price competition. On the other hand, the policy of granting a permissible delay adds not only an additional cost but also an additional dimension of default risk to vendors. In this paper, we will incorporate the fact that granting a permissible delay has a positive impact on demand but negative impacts on both costs and default risks to establish vendor–buyer supply chain models. Then we will derive the necessary and sufficient conditions to obtain the optimal solution for both the vendor and the buyer under non-cooperative Stackelberg equilibrium. Finally, we will use two numerical examples to show that (1) granting a permissible delay may significantly improve profits for both the vendor and the buyer, (2) the sensitivity analysis on the optimal solution with respect to each parameter, and (3) the comparisons between Nash and Stackelberg solutions.

1. Introduction

In practice, vendors usually offer their buyers a delay period in payment. During the period, there is no interest charge. Hence, buyers can earn the interest from sales revenue meanwhile vendors lose the interest earned during the same time. However, if the payment is not paid in full by the end of the permissible delay period, then vendors charge buyers an interest on the outstanding amount. The permissible delay in payment produces two benefits to the vendor: (1) it attracts new buyers who consider it to be a type of price reduction, and (2) it may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions. On the other hand, the policy of granting credit terms adds not only an additional cost but also an additional dimension of default risk to the vendor.

Goyal (1985) first developed an economic order quantity (EOQ) model for the buyer when the seller offers a fixed permissible delay period. Shah (1993) considered a stochastic inventory model when delays in payments are permissible. Jamal et al. (1997) extended Goyal's model to allow for shortages. Hwang and Shinn (1997) added the pricing strategy to the model, and derived the optimal price and lot sizing for a retailer under the condition of permissible delay in payments. Teng (2002) provided an alternative conclusion from Goyal (1985), and proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Shinn and Hwang (2003) considered both pricing and ordering policies under order-size dependent delay in payments. Chang et al. (2003) developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Huang (2003) extended Goyal's model to develop an EOQ model in which the supplier offers the retailer the up-stream trade credit period M, and the retailer in turn provides the down-stream trade credit period N (with $N \leq M$) to his/her customers. He then extended the EOQ model to the EPQ model in Huang (2004), and further generalized trade credit to conditional trade credit in Huang (2007), and then to partial trade credit in Huang and Hsu (2008). Ouyang et al. (2005) discussed an EOQ model for deteriorating items under trade credits, and then extended the model to allow for partial backlogging in Ouyang et al. (2006). Teng and Goyal (2007) complemented the shortcoming of Huang's model and proposed a generalized formulation. Goyal et al. (2007) established optimal ordering policies when the supplier provides a progressive interest-payable scheme. Liao (2008) studied deteriorating items under two-level trade credit. Chang et al. (2008)

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provided a review on inventory lot-size models under trade credits. Teng (2009) established an EOQ model for a retailer who offers distinct trade credits to its good and bad credit customers. Teng and Chang (2009) complemented the shortcoming in Huang (2007). Chang et al. (2010a) derived the optimal ordering policies for deteriorating items when a trade credit is linked to order quantity. Chang et al. (2010b) presented the optimal manufacturer's replenishment policies in a supply chain with up-stream and down-stream trade credits. Teng et al. (2011) obtained the retailer's optimal ordering policy when the supplier offers a progressive permissible delay in payments. Recently, Teng et al. (2012) generalized the demand pattern from constant to non-decreasing in time. Musa and Sani (2012) established inventory ordering policies of delayed deteriorating items under permissible delay in payments. Liao et al. (2012) studied lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit.

In all these articles described above, the inventory models are studied only from the perspective of the buyer whereas in practice the length of the credit period is set by the vendor. So far, how to determine the optimal length of the credit period for the vendor has received a very little attention by the researchers. Abad and Jaggi (2003) determined both the seller's and the buyer's policies under non-cooperative as well as cooperative relationships. However, in their model, the demand rate was not affected by offering a permissible delay. Jaggi et al. (2008) developed the optimal credit as well as replenishment policy jointly for the vendor when credit period has a positive impact on demand. However, Jaggi et al. (2008) only focused on the vendor, and did not study the interaction between the vendor and the buyer. Chern et al. (2013) developed the Nash equilibrium solution in a vendor–buyer supply chain model with permissible delay in payments. Lately, Su (2012) presented an optimal replenishment policy for an integrated inventory system with defective items and allowable shortage under trade credit. Recently, Liu and Cruz (2012) proposed supply chain networks with corporate financial risks and trade credits under economic uncertainty.

In this paper, we assume that the vendor is the dominating firm over the buyer. For instance, a vendor like Coca Cola has a dominant power over its local grocery store, and has the power to set its own strategy. As a result, the dominating firm (e.g., Coca Cola) acts like a leader. Conversely, its local grocery store (as a follower) simply follows the policy set by the dominating firm to derive its own strategy. Consequently, the non-cooperative Stackelberg equilibrium solution is usually applied in this situation. To fully understand the interaction between the vendor and the buyer in a non-cooperative Stackelberg business situation, we will develop an EOQ model for both the vendor and the buyer to reflect the real-life situations by incorporating the concepts of (1) vendors usually provide buyers a permissible delay in payment, (2) a vendor's credit period has a positive impact on its buyer's demand, and (3) the longer the credit period to the buyer, the higher the default risk as well as the cost to the vendor. We then will derive the necessary and sufficient conditions to obtain the optimal solution for both the vendor and the buyer under non-cooperative Stackelberg equilibrium. Finally, we will use two numerical examples to obtain (1) some managerial insights, and (2) comparisons between Nash and Stackelberg solutions.

2. Notation and assumptions

The following notation and assumptions are used in the paper. For convenience, subscripts v and b represent vendor and buyer respectively.

2.1. Notation

- S_i — the i's setup cost per production run (or per order), with $i=v$ and b.
- F — the vendor's fixed process cost to deal with each buyer's order.
- R — The vendor's annual production rate.
- C_i — the i's production cost (or purchasing cost) per unit, with $i=v$ and b.
- P_i — the i's unit selling price, with $i=v$ and b. We assume without loss of generality (WLOG) that $P_b > P_v = C_b > C_v$.
- H_i — the i's holding cost per unit per year excluding interest charge, with $i=v$ and b.
- I_e — the buyer's interest earned per dollar per year.
- I_c — the buyer's interest charged per dollar per year.
- r — the vendor's annual compounded interest rate on opportunity cost.
- m — the trade credit period in years offered by the vendor which is a real number (vendor's decision variable).
- n — the vendor's number of deliveries to the buyer per production cycle which is a positive integer (vendor's decision variable)
- $D=D(m)$ — the buyer's annual demand rate which is a function of m, with $D<R$.
- t — the buyer's replenishment cycle time in years which is a real number (buyer's decision variable).
- Q — the buyer's order quantity.
- TP_i — the i's annual total profit, with $i=v$ and b.
- m^* — the vendor's optimal credit period.
- n^* — the vendor's optimal number of deliveries per production cycle.
- t^* — the buyer's optimal replenishment cycle time.
- Q^* — the buyer's optimal order quantity $=Dt^*$.

2.2. Assumptions

Next, the following assumptions are made to establish the mathematical inventory model.

1. In today's global competition, many retailers have no pricing power. As a result, the selling price is hardly changed for many retailers. In addition, to avoid lasting price competition, we may assume WLOG that the selling price is constant in today's global competition and low inflation environment.
2. As stated in Jaggi et al. (2008), "it is observed that credit period offered by the retailer to its customers has a positive impact on demand of an item." For simplicity, we may assume that the buyer's demand rate $D(m)$ with the vendor's trade credit of m periods is given by

   $$D(m) = ke^{a\,m}, \text{ where } k \text{ and } a \text{ are positive constants.} \tag{1}$$

   For convenience, $D(m)$ and D will be used interchangeably.
3. Since the longer the credit period to the buyer, the higher the default risk, we may assume WLOG that the rate of not receiving debt obligations giving the credit period m is

   $$f(m) = 1-e^{-b\,m}, \text{ where } b \text{ is a positive constant.} \tag{2}$$

4. If $t \le m$, then the buyer pays the entire purchase amount to the vendor at time m. Otherwise, the buyer deposits sales revenue into an interest bearing account until m. At the end of the permissible delay, the retailer pays off all units sold, uses unsold items as collateral to apply for a loan, and pays the