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Some Stackelberg Type Location Game

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Abstract—This paper considers a Stackelberg type location game over the unit square $[0,1]\times[0,1]$. There are two chain stores, Players I and II, which sell the same kind of articles. Each store is planning to open a branch in region $[0,1]$. The purpose of each store is to decide the location to open its branch. In such a situation, the demand points, i.e., the customers, distribute continuously over $[0,1]$ in accordance with *cdf* $G(\cdot)$. Each customer wants to buy at a closer store between them, but never moves more than a distance ℓ. We also assume that Player I is forced to behave as the leader of this game and the opponent (Player II) is to be the follower. It is shown that there are various types of Stackelberg equilibriums according to the conditions of $G(\cdot)$ and ℓ. © 2003 Elsevier Ltd. All rights reserved.

Keywords—Nonzero sum game, Location, Infinite game, Stackelberg equilibrium.

1. INTRODUCTION

This paper considers a Stackelberg type location game over unit square $[0,1]\times[0,1]$. The model is described as follows.

There are two chain stores which deal in the same kind of articles. Each of the two stores is planning to open a branch at some region where there are no such stores now. The region takes the shape of a line segment, so that we represent it as the unit interval $[0,1]$. The purpose of each store is to decide the location to open its branch in $[0,1]$. Though both stores duopolize this new market over $[0,1]$, it is natural to assume the possibility to open at the same time by both stores is negligible. Thus, one of the two stores is forced to behave as the leader of this game, while on the other hand the opponent is to be the follower. We have to consider a Stackelberg

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type location game. We call the leader Player I and the follower Player II, respectively. Under the above situations, the demand points, that is customers, distribute continuously over $[0,1]$ in accordance with *cdf* $G(\cdot)$. Each of the customers wants to buy at the closer store between them, but never moves more than a certain distance ℓ $(0 < \ell < 1)$. Each player has to decide the optimal location from the viewpoint of Stackelberg equilibrium.

Here, we summarize the assumptions and notations to show our model explicitly as follows.

(i) The customers (demand points) distribute continuously over the line segment $[0,1]$ in accordance with *cdf* $G(\cdot)$ which has its *pdf* $g(\cdot)$.

(ii) When a store locates at point $z \in [0,1]$, a customer who lives at point $t \in [0,1]$ goes to buy at this store with probability $u(t \mid z)$.

(iii) Each customer usually wants to buy at the closest store in $[0,1]$.

Under the assumptions mentioned above, Player I locates first his branch at point $x \in [0,1]$. Then Player II decides the location $y \in [0,1]$ of his branch after observing the location x of his opponent. That is, Player I is the leader and Player II is the follower in this game. Each player has to decide the location of his branch which maximizes to obtain the number of customers in the interval $[0,1]$ on the steady state from the viewpoint of Stackelberg equilibrium, at their planning stage. When both players locate their branches at the same position in $[0,1]$, Players I and II share the market between them with even ratio.

Related to this game, Hotelling first pointed out and considered the location problem from the viewpoint of stability in competition between two players in 1929 [1]. After that, much research extended his work from a game theoretical viewpoint (for example, [2–7]). Gabszewicz and Thisse [8] summarized an excellent survey in *Handbook of Game Theory*. But, they analyzed and considered Nash equilibrium for the location game but not Stackelberg type. Osumi *et al.* proposed and analyzed competitive facility location models but not exactly game theoretical [9,10].

2. GENERAL FORMULATION

Let $M_i(x,y)$ be the expected payoff to Player i $(i = 1,2)$ when Players I and II locate their branches at points x and y in $[0,1]$, respectively. We have

$$M_1(x,y) = \begin{cases} \int_0^{(x+y)/2} u(t \mid x)\, g(t)\, dt, & x < y, \\ \frac{1}{2} \int_0^1 u(t \mid x)\, g(t)\, dt, & x = y, \\ \int_{(x+y)/2}^1 u(t \mid x)\, g(t)\, dt, & x > y, \end{cases} \tag{1}$$

$$M_2(x,y) = \begin{cases} \int_0^{(x+y)/2} u(t \mid y)\, g(t)\, dt, & y < x, \\ \frac{1}{2} \int_0^1 u(t \mid y)\, g(t)\, dt, & y = x, \\ \int_{(x+y)/2}^1 u(t \mid y)\, g(t)\, dt, & y > x. \end{cases} \tag{2}$$

Here, we establish the pure strategy for each player. Since this game is a nonzero sum infinite game between the leader (Player I) and the follower (Player II), it is natural to define $x \in [0,1]$ as the pure strategy for Player I and $y(x) \in [0,1]$ as the pure strategy for Player II. And the purposes of two players have to decide $y^*(x)$ and x^* which satisfy the following two stage maximization process.

Since Player II is the follower of this game, he can maximize his playoff $M_2(x, y(x))$ by selecting strategy $y(x)$ after observing x of Player I. On the other hand, Player I is the leader and knows the payoff functions of both players $M_1(x,y)$, $M_2(x,y)$, and hence, he learns Player II's set of best responses $\{y^*(x) \mid M_2(y^*(x)) = \sup_y M_2(x,y)\}$ to any strategy x of Player I. Having this information he then maximizes his payoff by choosing x^* from condition $M_1(x^*) = \sup_x M_1(x, y^*(x))$. Thus, the situation $(x^*, y^*(x^*))$ is an equilibrium point which gives the Stackelberg equilibrium.

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