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Stochastic Stackelberg equilibria with applications to time-dependent newsvendor models

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ABSTRACT

In this paper, we prove a maximum principle for general stochastic differential Stackelberg games, and apply the theory to continuous time newsvendor problems. In the newsvendor problem, a manufacturer sells goods to a retailer, and the objective of both parties is to maximize expected profits under a random demand rate. Our demand rate is an Itô–Lévy process, and to increase realism information is delayed, e.g., due to production time. A special feature of our time-continuous model is that it allows for a price-dependent demand, thereby opening for strategies where pricing is used to manipulate the demand.

1. Introduction

The one-period newsvendor model is a widely studied object that has attracted increasing interest in the last two decades. The basic setting is that a retailer wants to order a quantity q from a manufacturer. Demand D is a random variable, and the retailer wishes to select an order quantity q maximizing his expected profit. When the distribution of D is known, this problem is easily solved. The basic problem is very simple, but appears to have a never-ending number of variations. There is now a very large literature on such problems, and for further reading we refer to the survey papers by Cachón (2003) and Qin et al. (2011) and the numerous references therein.

The (discrete) multiperiod newsvendor problem has been studied in detail by many authors, including Matsuyama (2004), Berling (2006), Bensoussan et al. (2007, 2009), Wang et al. (2010), just to quote some of the more recent

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Nomenclature

Main variables

w	wholesale price per unit (chosen by the manufacturer)
q	order quantity (rate chosen by the retailer)
R	retail price per unit (chosen by the retailer)
D	demand (random rate)
M	production cost per unit (fixed)
S	salvage price per unit (fixed)

contributions. Two papers whose approach is not unlike that used in our paper are Kogan (2003) and Kogan and Lou (2003), where the authors consider continuous time-scheduling problems.

In many cases, demand is not known and the parties gain information through a sequence of observations. There is a huge literature on cases with partial information, e.g., Scarf (1958), Gallego and Moon (1993), Bensoussan et al. (2007), Perakis and Roels (2008), Wang et al. (2010), just to mention a few. When a sufficiently large number of observations have been made, the distribution of demand is fully revealed and can be used to optimize order quantities. This approach only works if the distribution of D is static, and leads to false conclusions if demand changes systematically over time. In this paper we will assume that the demand rate is a stochastic process D_t and we seek optimal decision rules for that case.

In our paper, a retailer and a manufacturer write contracts for a specific delivery rate following a decision process in which the manufacturer is the leader who initially decides the wholesale price. Based on that wholesale price, the retailer decides on the delivery rate and the retail price. We assume a Stackelberg framework, and hence ignore cases where the retailer can negotiate the wholesale price. The contract is written at time $t-\delta$, and goods are received at time t. It is essential to assume that information is delayed. If there is no delay, the demand rate is known, and the retailer's order rate is made equal to the demand rate. Information is delayed by a time δ. One justification for this is that production takes time, and orders cannot be placed and effectuated instantly. It is natural to think about δ as a production lead time.

The single period newsvendor problem with price dependent demand is classical, see Whitin (1955). Mills (1959) refined the construction considering the case where demand uncertainty is added to the price-demand curve, while Karlin and Carr (1962) considered the case where demand uncertainty is multiplied with the price-demand curve. For a nice review of the problem with extensions see Petruzzi and Dada (1999). Stackelberg games for single period newsvendor problems with fixed retail price have been studied extensively by Lariviere and Porteus (2001), providing quite general conditions under which unique equilibria can be found.

Multiperiod newsvendor problems with delayed information have been discussed in several papers, but none of these papers appears to make the theory operational. Bensoussan et al. (2009) use a time-discrete approach and generalize several information delay models. However, these are all under the assumption of independence of the delay process from inventory, demand, and the ordering process. They assert that removing this assumption would give rise to interesting as well as challenging research problems, and that a study of computation of the optimal base stock levels and their behavior with respect to problem parameters would be of interest. Computational issues are not explored in their paper, and they only consider decision problems for inventory managers, disregarding any game theoretical issues.

Calzolari et al. (2011) discuss filtering of stochastic systems with fixed delay, indicating that problems with delay lead to nontrivial numerical difficulties even when the driving process is Brownian motion. In our paper, solutions to general delayed newsvendor equilibria are formulated in terms of coupled systems of stochastic differential equations. Our approach may hence be useful also in the general case where closed form solutions cannot be obtained.

Stochastic differential games have been studied extensively in the literature. However, most of the works in this area have been based on dynamic programming and the associated Hamilton–Jacobi–Bellman–Isaacs type of equations for systems driven by Brownian motion only. More recently, papers on stochastic differential games based on the maximum principle (including jump diffusions) have appeared. See, e.g., Øksendal and Sulem (2012) and the references therein. This is the approach used in our paper, and as far as we know, the application to the newsvendor model is new. The advantage with the maximum approach is two-fold:

- We can handle non-Markovian state equations and non-Markovian payoffs.
- We can deal with games with partial and asymmetric information.

Fig. 1 shows a sample path of an Ornstein–Uhlenbeck process that is mean reverting around a level $\mu=100$. Even though the long-time average is 100, orders based on this average are clearly suboptimal. At, e.g., $t=30$, we observe a demand rate $D_{30}=157$. When the mean reversion rate is as slow as in Fig. 1, the information $D_{30}=157$ increases the odds that the demand rate is more than 100 at time $t=37$. If the delay $\delta=7$ (days), the retailer should hence try to exploit this extra information to improve performance.

Based on the information available at time $t-\delta$, the manufacturer should offer the retailer a price per unit w_t for items delivered at time t. Given the wholesale price w_t and all available information, the retailer should decide on an order rate q_t and a retail price R_t. The retail price can in principle lead to changes in demand, and in general the demand rate D_t is, hence, a function of R_t. However, such cases are hard to solve in terms of explicit expressions. We will also look at the simplified case where R is exogenously given and fixed. To carry out our construction, we will need to assume that items cannot be stored. That is of course a strong limitation, but applies to important cases like electricity markets and markets for fresh foods.